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METHODS OF STANDARDIZING ICE-CREAM MIXES

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Standardizing an ice-cream mix means calculating the relative amounts of ingredients required for a finished product of a desired composition. The calculations are made for a definite weight of ice-cream mix, the exact amount depending upon the size of the standard batch processed in a given plant. For simplicity, however, the calculations made here will usually be for a 100-pound batch, since that amount offers a satisfactory basis for making any desired quantity of mix.

Errors often occur in making mixes. These commonly result from careless weighing of products, or from using ingredients with a different composition from that assumed during the original calculations, or, in some instances, from faulty calculations. Provided the errors are great enough, a recalculation is necessary, to ascertain how the original mixture must be modified to assure the desired composition. This process is termed *restandardization.*

A satisfactory mix for plain as well as for fruit ice creams will ordinarily contain 10 to 14 per cent fat, 9 to 11 per cent milk-solids-not-fat, 15 to 16 per cent sugar, and sufficient stabilizer to insure a smooth-textured finished product. In California the fat contained in 100 pounds of legal vanilla ice cream (see p. 3), would satisfy the requirements for 125 pounds of fruit ice cream—a dilution of 25 per cent. If a higher dilution is desired, the basic mix would have to contain more than the minimum of 10 per cent fat. Often the standard adopted is considerably higher than the legal requirements.

This circular is intended to aid commercial ice-cream manufacturers or others interested in making the computations necessary for propor-

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tioning the ingredients in ice-cream mixes. Not all the available methods for standardization and restandardization are considered in this publication; but there are definite reasons for including the methods given, as can be seen from the brief advance discussion that follows.

1. Ordinarily the Pearson-square method is as simple as any. With this method no formulas need be memorized. The computations, however, are comparatively long, especially when used to standardize a complex mix such as one requiring plain condensed or sweetened condensed milk.

2. The serum-point or formula method is shorter than the Pearson square. It can almost always be used and does not become too complicated for the solution of complex mixes. Though its formulas appear rather complicated, they are easily learned.

3. The main advantages of the algebraic method are accuracy and general applicability. Usually, however, it is longer than the serum-point method and, as ordinarily used, scarcely more accurate; besides, it is often more difficult to check if an error is introduced in the calculations.

4. The graphic method, although not so common, is fairly simple and is accurate enough for practical purposes. Its main advantage seems to lie in its easy application to restandardization problems. Those accustomed to graphs should find it convenient and satisfactory.

5. Tabulated recipes (table 2, p. 14), for ice cream and ice milk on a 100-pound basis, will be sufficient to meet the needs of many ice-cream makers.

In table 2 the most difficult parts of the calculations have been completed. All that remains is to ascertain the correct proportion of milk and cream to insure the indicated milk-fat content for each milk and cream mixture listed. Ordinarily, this calculation is most conveniently made by means of the Pearson square.

The particular method employed in restandardization will depend upon the type of problem. Examples given in a later section will illustrate the applications of several methods.

DEFINITIONS

Some common terms applied to standardization and restandardization will be defined here to prevent confusion and to aid in clarifying the explanations, illustrations, and formulas that follow. For certain terms that have acquired rather specialized meanings because of continued use in this field, it seems better to give the accepted meaning rather than modify the terminology.

Ice cream is a frozen food often consumed as a dessert. Though essentially a milk product, it also contains sugar, flavoring, and sometimes

other constituents. The various state laws all have minimum fat standards for ice cream, varing from 8 per cent to 14 per cent. According to the California Agricultural Code,[2] plain or vanilla ice cream must contain a minimum of 10 per cent milk fat; fruit or nut ice cream, 8 per cent. There must, furthermore, be a minimum of 1.6 pounds of food solids to the gallon.

Ice milk is a frozen product resembling ice cream except that it contains considerably less milk fat. It is often used in manufacturing chocolate-coated frozen confections or in preparing milk shakes and other fountain drinks. According to the California Agricultural Code, ice milk must contain at least 4 per cent milk fat and at least 1.3 pounds of food solids per gallon.

A *mix* is a mixture of ingredients that when frozen is converted into ice cream, ice milk, or similar products. As ordinarily used it does not include added flavoring, but in certain cases the flavoring may be a part of the standardized mix.

Off batch is a term often applied to a batch of mix that lacks the desired composition. In restandardizing, the off batch is adjusted or corrected to the composition desired by the addition of the necessary amount of one or more other ingredients.

Nonmilk products are the ingredients that do not originate from milk. They include sucrose, dextrose, and similar sweetening agents; gelatin, sodium alginate, gums, and other stabilizers; egg yolk in several different forms; flavoring materials sometimes used as part of the mix; water other than that contained in the milk products themselves; and certain other products occasionally added in standardizing mixes. Some of these constituents may be combined with milk products, and this mixture added as an ingredient of the mix. Sugar, for example, is present in sweetened condensed milk or is commonly added to cream before freezing for storage. Such sugar, even though added along with milk products, is considered a nonmilk product. Similarly, other nonmilk products remain so even though they may be first added to milk products and then used as mix ingredients.

Milk products are, obviously, all derived from milk. This classification includes milk, cream, and skim milk in their natural forms or in special forms resulting from processing, such as the removal of all or part of the water or the development of special characteristics because of controlled heat treatment, or freezing, or the further separation of any product into its respective components. One must remember also that when nonmilk products are added to milk products during processing,

[2] For further details regarding the California standard, see: State of California Agricultural Code 1941, Division 4, Chapter 2, Article 4, Sections 506–15. Issued by the Supervisor of Documents, Sacramento, California. 1941.

as in sweetened condensed milk, the finished product is composed of a nonmilk-products fraction and a milk-products fraction.

Serum, as used in standardizing ice-cream mixes, usually refers to milk products exclusive of their fat content. More correctly, the word would be *plasma,* according to accepted usage in other fields. Strictly speaking, milk serum should be defined as the clear liquid obtained from milk by freeing it from fat, proteins, and colloidally dispersed minerals. In the ice-cream industry, *plasma* and *serum* are synonymous, although the latter is the more common term. In this publication, because of the accepted practice in the industry, *serum* will refer to the fat-free portion of milk products—that is, milk products minus fat. Accordingly,

mix serum = milk products in mix — milk fat in mix.

Milk-solids-not-fat and *serum solids* are synonymous terms to designate the solids contained in serum. In other words, they represent the solids remaining after the fat and water have been removed from any milk product. Some may prefer the term *plasma solids* to either of those suggested.

Milk-solids-not-fat concentrate and *serum-solids concentrate* are used synonymously to designate products containing milk-solids-not-fat or serum solids in greater concentration than they normally exist in skim milk. Such a product may or may not contain added nonmilk products. In any case, except where a mix is to be processed in a vacuum pan, if it is to be a satisfactory mix ingredient, it must contain milk-solids-not-fat in sufficient concentration so that by properly proportioning the available ingredients one may obtain the desired composition. Obviously, therefore, the concentration of serum solids in the serum-solids concentrate must exceed that desired in the final mix.

Overrun in ice cream is the increase in volume of ice cream as compared with the volume of mix and flavoring used. It is usually expressed as per cent overrun. One may calculate it by considering the increase in volume of ice cream per unit weight as compared with the volume of the same weight of mix, or by considering the decrease in weight per unit volume of ice cream as compared with the weight of the same volume of mix. Formulas illustrating both methods appear later in this publication.

The term *stabilizers,* as applied to the manufacture of ice cream and related products, refers to such ingredients as gelatin, sodium alginate, pectin, certain gums, and similar substances, which because of their colloidal properties may retard the development of relatively large ice crystals during storage. Ice cream is sometimes made without any added stabilizer, but its smooth texture usually depends in part upon its stabilizer content.

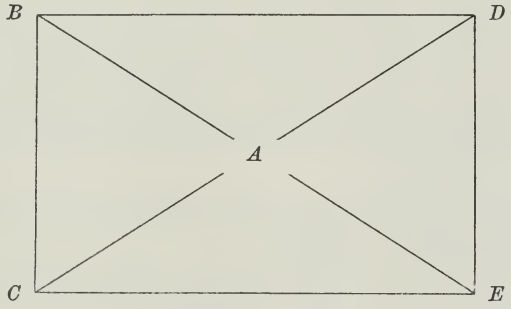
METHODS OF MIX STANDARDIZATION

PEARSON-SQUARE METHOD

The Pearson-square method, rather generally used in standardizing milk and cream for fat content, will serve equally well in standardizing mixes for milk fat and for milk-solids-not-fat. Here, however, one must make certain arbitrary divisions of the mix to facilitate calculation.

In this method a square or rectangle serves as a guide to the necessary steps in calculating the relative amounts of two ingredients required for a mixture of desired composition. Obviously the percentage composition of one ingredient must be above and that of the other must be below the desired value in the final mixture.[3]

Before consideration of specific problems, the following working diagram is given to illustrate the basis of this method:



A = Percentage of the desired constituent (that is, milk fat or milk-solids-not-fat) in the final mixture.

B = Percentage of this constituent in one of the ingredients.

C = Percentage of this constituent in the other ingredient.

D = Unit parts of the ingredient in the final mixture containing the B percentage; D, a positive number, equals the difference between C and A.

E = Unit parts of the ingredient in the final mixture containing the C percentage; E, a positive number, equals the difference between A and B.

The total unit parts of the final mixture having A percentage of the desired constituent equals the sum of D and E. Consequently,

$$\left.\begin{array}{l}\text{the fraction of the total mixture which will be the ingredient with } B \text{ percentage}\end{array}\right\} = \frac{D}{D+E}, \text{ and}$$

$$\left.\begin{array}{l}\text{the weight of the ingredient with } B \text{ percentage}\end{array}\right\} = \left[\frac{D}{D+E}\right] \times \left[\begin{array}{c}\text{total weight of} \\ \text{the desired mixture}\end{array}\right].$$

[3] It is possible also to use the Pearson square as a basis of calculating the amount of skim milk or cream that must be removed from a known quantity of milk in order to insure the desired composition in the portion remaining. This application of the method will not be discussed here, although it may be used advantageously in the cheese industry and under certain conditions in the evaporated-milk industry.

Likewise,

$$\left.\begin{array}{l}\text{the fraction of the total mixture which will be the ingredient with } C \text{ percentage}\end{array}\right\} = \frac{E}{D+E}, \text{ and}$$

$$\left.\begin{array}{l}\text{the weight of the ingredient with } C \text{ percentage}\end{array}\right\} = \left[\frac{E}{D+E}\right] \times \left[\begin{array}{c}\text{total weight of} \\ \text{the desired mixture}\end{array}\right].$$

The solution to the following problem illustrates the use of this method in standardizing for milk-solids-not-fat and for milk fat. In all problems of this type one must first standardize for the milk-solids-not-fat in the serum of the mix. Even though there are more than two sources of milk-solids-not-fat, the products can ordinarily be grouped so that standardization is accomplished by considering only two sources. The explanation is that the skim-milk portions of both milk and cream are "normal" skim milk and for practical purposes both may be considered to contain the same percentages of milk-solids-not-fat.

Problem: Suppose we desire to prepare 100 pounds of mix containing 12.0 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.5 per cent gelatin. Consider the following products to be available: cream testing 40.0 per cent fat, milk testing 4.0 per cent fat, condensed skim milk containing 30.0 per cent milk-solids-not-fat, and the nonmilk products, sugar and gelatin.

The steps in the calculations are given below, and the results are verified in tabulated check 1 on page 9.

For convenience in calculating we can divide the mix into *nonmilk products* and *milk products*. The milk-products portion can further be subdivided into *milk fat* and *serum*. The serum in turn contains *milk-solids-not-fat* and water. Let us apply these facts to the problem stated above.

Since the sugar and gelatin in this case are each obtained from one source, we calculate their respective weights by multiplying the weight of the mix by the fraction of each desired. Thus

$100 \times 0.15 = 15.0 =$ pounds of sugar; and
$100 \times 0.005 = 0.5 =$ pounds of gelatin,
or a total of $15.5 =$ pounds of nonmilk products.

Hence 100 pounds of mix minus 15.5 pounds of nonmilk products leaves 84.5 pounds of milk products.

If we temporarily disregard the pounds of fat in the mix, the first part of the problem resolves itself into determining the percentage of milk-solids-not-fat in the serum and then using this value in the Pearson square, as indicated below. Thus 84.5 pounds of milk products minus 12.0 pounds of milk fat equals 72.5 pounds of mix serum, which contains 10.0 pounls of milk-solids-not-fat. It is necessary to ascertain the average

percentage of milk-solids-not-fat in the serum of the mix in order to use this method. This value can be calculated as follows:

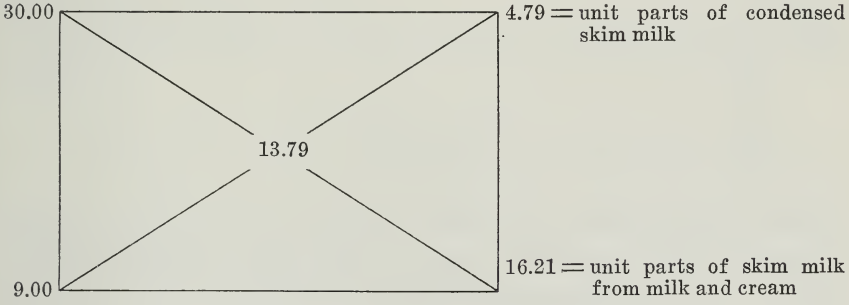
$$\text{Average percentage of milk-solids-not-fat in the serum of the mix} = \frac{\text{weight of milk-solids-not-fat in the serum of the mix}}{\text{weight of serum of the mix}} \times 100.$$

Hence

$$\frac{10.0}{72.5} \times 100 = 13.79 = \text{the average percentage of milk-solids-not-fat in the serum of the mix.}$$

According to these calculations, the serum of the mix must contain 13.79 per cent of milk-solids-not-fat. The products available from which to supply the serum of the mix with its required percentage of serum solids are condensed skim milk testing 30.00 per cent milk-solids-not-fat, and the skim-milk portion of the milk and cream used. The latter can for all practical purposes be considered to contain 9.00 per cent serum solids. If in any specific case the composition of the skim milk deviates from this value, the correct figure can be substituted in the calculations. Thus, if the skim milk contained 8.80 per cent milk-solids-not-fat, use 0.088 instead of 0.09. (For other values, see table 3.)

The procedure already outlined on pages 5 and 6 may be followed, with numbers substituted for the letters used in the general example. Thus



Then $4.79 + 16.21 = 21.00 =$ the unit parts of the mixture testing 13.79 per cent serum solids.

Hence $\frac{4.79}{21.00} \times 72.5 = 16.54 =$ pounds of condensed skim milk required.

Similarly, the pounds of skim milk could be calculated, but this substance is supplied by the milk and cream and is called "skim milk" only for purposes of computation. Evidently, therefore, if the milk and cream are correctly proportioned they must contain the required amount of skim milk. The next calculations will deal, therefore, with the proportioning of the milk and cream to supply the milk-fat requirements of the mix.

One can accomplish this purpose by considering the milk fat of the

mix, temporarily eliminated in the previous calculations. The average percentage of fat in the milk and cream must be evaluated and used in the rectangle as indicated below. Thus

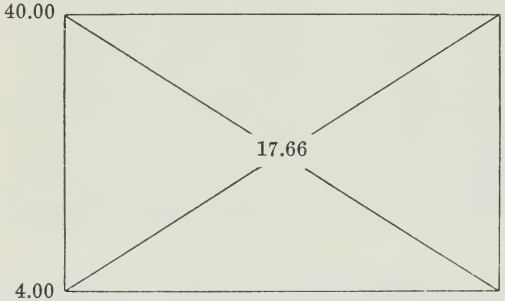
$$84.50 - 16.54 = 67.96 = \text{pounds of milk and cream desired.}$$

The average percentage of fat in the milk and cream is calculated as follows:

$$\text{Average percentage of fat in the milk and cream} = \frac{\text{weight of fat in the milk and cream}}{\text{weight of milk and cream}} \times 100.$$

$$\text{Hence } \frac{12.0}{67.96} \times 100 = 17.66 \text{ per cent fat in the milk and cream mixture.}$$

The Pearson-square method is again used, to calculate the weight of milk and cream required. Thus



13.66 = unit parts of cream testing 40.0 per cent fat

22.34 = unit parts of milk testing 4.0 per cent fat

Then $13.66 + 22.34 = 36.00 =$ unit parts of mixture containing 17.66 per cent fat;

$\frac{13.66}{36.00} \times 67.96 = 25.79 =$ pounds of cream; and

$\frac{22.34}{36.00} \times 67.96 = 42.17 =$ pounds of milk.

One can also calculate the weight of milk by subtracting the weight of cream already determined from the total weight of milk and cream. Thus

$$67.96 - 25.79 = 42.17 = \text{pounds of milk.}$$

The foregoing calculations are verified in tabulated check 1.

The mix just illustrated would be considered a simple mix. This method is satisfactory for such calculations and, in addition, can often be profitably used in conjunction with the serum-point method that will be illustrated next. Even though it can be used in complex mixes, however, the calculations required for standardization are unnecessarily lengthened when the serum-solids concentrate used contains fat, or fat and sugar in addition to milk-solids-not-fat. For this reason its use will not be illustrated further except as a means of standardizing the fat content of the milk and cream mixtures in tables 1 and 2 (p. 13 and 14),

and of mixes made in a vacuum pan; or as an optional method for part of the calculations in connection with the graphic method of standardization.

SERUM-POINT, OR FORMULA, METHOD

One can, by simple algebra, set up an equation for milk-solids-not-fat and another for the milk fat of the mix and from these derive general working formulas for solving most standardization problems. A set of such formulas is given below. In these, the composition of skim milk, or

TABULATED CHECK 1

Ingredients used	Weight	Fat supplied	Milk-solids-not-fat supplied	Total solids supplied
	pounds	*pounds*	*pounds*	*pounds*
Condensed skim milk testing 30.0 per cent milk-solids-not-fat	16.54	.....	4.96	4.96
Cream testing 40.0 per cent fat	25.79	10.32	1.39	11.71
Milk testing 4.0 per cent fat	42.17	1.69	3.64	5.33
Sugar	15.00	.....	.....	15.00
Gelatin	0.50	.....	.....	0.50
Totals supplied	100.00	12.01	9.99	37.50
Totals required	100.00	12.00	10.00	37.50

the skim-milk portion of milk and cream, is taken as 9.0 per cent milk-solids-not-fat, which, as already explained, represents a safe average figure. Let

X = pounds of cream with a fraction fat,
Y = pounds of milk with b fraction fat,
Z = pounds of serum-solids concentrate[4] with c fraction fat, d fraction milk-solids-not-fat, and e fraction serum,
S = pounds of serum in the mix (milk products minus milk fat) containing R pounds of serum solids,
F = pounds of milk fat in the mix, and
P = pounds of milk and cream; that is, the quantity $(X + Y)$.

Then we obtain the following equation for serum solids:

$$R = dZ + 0.09\ (S - eZ).$$

The formula for calculating the weight of serum-solids concentrate is obtained by solving this equation for Z. Thus

$$Z = \frac{R - 0.09S}{d - 0.09e}.$$

[4] Z can be condensed skim or whole milk, either sweetened or unsweetened; or whole or skim-milk powder; or, for that matter, any product containing milk-solids-not-fat in greater concentration than is present in normal milk serum (that is, skim milk).

Likewise, the following equation for milk fat may be obtained:

$$F = aX + bY + cZ.$$

And the formula for calculating the weight of cream is secured by solving this equation for X, at the same time substituting P for the quantity $(X + Y)$. Thus

$$X = \frac{F - cZ - bP}{a - b}.$$

The following equation can then be used for calculating the weight of milk:

$$Y = P - X.$$

It may clarify these formulas somewhat to represent them, respectively, in the following manner (with the addition of the formulas for calculating nonmilk products):

$$\text{(1) Pounds of serum-solids concentrate} = \frac{\left[\begin{array}{l}\text{pounds of milk-}\\ \text{solids-not-fat in}\\ \text{the mix}\end{array}\right] - \left[0.09 \times \text{pounds of mix serum}\right]}{\left[\begin{array}{l}\text{fraction of milk-}\\ \text{solids-not-fat in}\\ \text{the serum-solids}\\ \text{concentrate}\end{array}\right] - \left[0.09 \times \begin{array}{l}\text{fraction of serum}\\ \text{in the serum-solids}\\ \text{concentrate}\end{array}\right]}.$$

$$\text{(2) Pounds of cream} = \frac{\left[\begin{array}{lll}\text{total} & & \text{pounds fat}\\ \text{pounds} & - & \text{in serum-}\\ \text{of fat} & & \text{solids}\\ \text{in mix} & & \text{concentrate}\end{array}\right] - \left[\begin{array}{lll}\text{fat} & & \text{pounds}\\ \text{fraction} & \times & \text{of milk}\\ \text{of} & & \text{and}\\ \text{milk} & & \text{cream}\end{array}\right]}{\text{fraction of fat in cream} - \text{fraction of fat in milk}}.$$

(3) Pounds of milk = pounds of milk and cream minus pounds of cream.

(4) Pounds of sugar = pounds of sugar desired in the mix minus pounds of sugar added in serum-solids concentrate or cream.

(5) Pounds of gelatin = weight of mix × fraction of gelatin in mix.

This method uses the divisions and subdivisions of the mix already illustrated in the Pearson-square method. Considering the mix from this point of view, one can calculate the correct proportion of ingredients for mixes irrespective of whether they are simple or complex. In other words, the basis of the calculation is the same whether the fat is furnished by one or by several ingredients and whether or not one of the dairy products used contains sugar.

First let us use the method to standardize a mix where the serum-solids concentrate contains no fat and no sugar. This mix resembles the one just illustrated, but we will modify it slightly to include egg yolk (a nonmilk product).

Problem: Suppose we wish to standardize 1,800 pounds of mix to contain 14.0 per cent milk fat, 9.5 per cent milk-solids-not-fat, 15.0 per cent sugar, 0.5 per cent egg-yolk solids, and 0.3 per cent gelatin. Consider the following products to be available: sugar, gelatin, egg yolk

containing 50 per cent total solids, cream containing 36.0 per cent fat, milk containing 3.7 per cent fat, and skim-milk powder containing 97.0 per cent total solids which are equal to serum solids.

The pounds of nonmilk products can be calculated as follows:

$$1{,}800 \times 0.15 = 270.0 = \text{pounds of sugar; and}$$
$$1{,}800 \times 0.003 = 5.4 = \text{pounds of gelatin; and}$$
$$1{,}800 \times 0.005 = 9.0 = \text{pounds of egg-yolk solids}$$
$$\text{or } \frac{9.0}{0.50} = 18.0 = \text{pounds of egg yolk.}$$

Hence $270.0 + 5.4 + 18.0 = 293.4 =$ pounds of nonmilk products in the mix.

Therefore, 1,800 pounds of mix minus 293.4 pounds of nonmilk products leaves 1,506.6 pounds of milk products. Since there must be $1{,}800 \times 0.14 = 252.0 =$ pounds of milk fat in the mix, there would be 1,506.6 pounds milk products minus 252.0 pounds of milk fat equals 1,254.6 pounds of mix serum, which must contain $1{,}800 \times 0.095 = 171.0 =$ pound of serum solids.

These data can now be used to calculate the weight of milk products for the mix by making the proper substitutions in the formulas just given.

$$(1)\ \text{Pounds of skim-milk powder} = \frac{171.0 - (0.09 \times 1{,}254.6)}{0.97 - 0.09} = 66.01.$$

Now the pounds of milk and cream in the mix can be ascertained by subtracting the weight of skim-milk powder from the total weight of milk products as follows:

$1{,}506.60 - 66.01 = 1{,}440.59 =$ pounds of milk and cream.

$$(2)\ \text{Pounds of cream} = \frac{252.0 - (0.037 \times 1{,}440.59)}{0.360 - 0.037} = 615.17.$$

(3) Pounds of milk $= 1{,}440.59 - 615.17 = 825.42.$

Since there is only one source for each of the nonmilk products, the calculations already made will apply. Tabulated check 2 verifies the calculations.

To test this method in the solution of a more complex standardization, consider the following problem. As can be seen, the same formulas apply whether the serum-solids concentrate contains only milk-solids-not-fat or whether it contains, in addition, milk fat or sugar, or both.

Problem: Suppose 100 pounds of mix must contain 13.0 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.4 per cent gelatin. Consider the following products available: sugar, gelatin, cream containing 40.0 per cent fat, milk containing 4.0 per cent fat, and sweetened condensed whole milk containing 8.0 per cent fat, 20.0 per cent milk-solids-not-fat, and 42.0 per cent sugar.

The mix must contain 15.4 pounds of nonmilk products and 84.6 pounds of milk products, of which 13 pounds must be milk fat. This leaves 71.6 pounds of mix serum, which in turn must contain 10 pounds of milk-solids-not-fat. The calculations for this standardization are complicated by the fact that the serum-solids concentrate contains milk

TABULATED CHECK 2

Ingredients used	Weight	Fat supplied	Milk-solids-not-fat supplied	Total solids supplied
	pounds	*pounds*	*pounds*	*pounds*
Skim-milk powder testing 97.0 per cent total solids	66.01	.....	64.03	64.03
Cream testing 36.0 per cent fat	615.17	221.46	35.43	256.89
Milk testing 3.7 per cent fat	825.42	30.54	71.54	102.08
Sugar	270.00	.....	.....	270.00
Egg yolk	18.00	.....	.....	9.00
Gelatin	5.40	.....	.....	5.40
Totals supplied	1,800.00	252.00	171.00	707.40
Totals required	1,800.00	252.00	171.00	707.40

TABULATED CHECK 3

Ingredients used	Weight	Milk fat supplied	Milk-solids-not-fat supplied	Sugar supplied	Total solids supplied
	pounds	*pounds*	*pounds*	*pounds*	*pounds*
Condensed whole milk testing 8.0 per cent fat, 20.0 per cent milk-solids-not-fat, 42.0 per cent sugar	22.94	1.84	4.59	9.63	16.06
Cream testing 40.0 per cent fat	23.09	9.24	1.25	....	10.49
Milk testing 4.0 per cent fat	48.20	1.93	4.16	....	6.09
Sugar	5.37	....	....	5.37	5.37
Gelatin	0.40	....	....	....	0.40
Totals supplied	100.00	13.01	10.00	15.00	38.41
Totals required	100.00	13.00	10.00	15.00	38.40

fat, milk-solids-not-fat, and sugar—a nonmilk product. Indeed, only 50 per cent of the serum-solids concentrate is serum.[5] By taking this into account and applying the formulas given, we obtain the desired results. Thus

$$(1)\ \text{Pounds of condensed milk} = \frac{10.00 - (0.09 \times 71.6)}{0.20 - (0.09 \times 0.50)} = 22.94.$$

[5] The fraction of serum in the serum-solids concentrate is calculated by subtracting from the whole the fraction of fat and the fraction of added sugar. Thus in this case 1.00 minus (0.08 fat plus 0.42 sugar) equals 0.50 serum. This equals the value of e in formula 1.

(2) Pounds of cream $= \frac{13.00 - (0.08 \times 22.94) - (0.04 \times 71.29)}{0.40 - 0.04} = 23.09.$

(3) Pounds of milk $= 71.29 - 23.09 = 48.20.$

(4) Pounds of sugar $= 15.00 - (22.94 \times 0.42) = 5.37.$

(5) Pounds of gelatin $= 100 \times 0.004 = 0.40.$

Tabulated check 3 shows the figures to be correct.

TABLE 1

Ingredients Required for Batches of Ice-Cream Mix Containing 12.0 Per Cent Fat, 10.5 Per Cent Milk-Solids-Not-Fat, 15.0 Per Cent Sugar, and 0.35 Per Cent Gelatin

Mix	Standardized milk and cream mixture testing 17.8 per cent fat*	Condensed skim milk testing 32.0 per cent milk-solids-not-fat	Sugar	Gelatin
pounds	*pounds*	*pounds*	*pounds*	*pounds*
100	67.43	17.22	15.00	0.35
200	134.86	34.44	30.00	0.70
300	202.29	51.66	45.00	1.05
400	269.72	68.88	60.00	1.40
500	337.15	86.10	75.00	1.75
600	404.58	103.32	90.00	2.10
700	472.01	120.54	105.00	2.45
800	539.44	137.76	120.00	2.80
900	606.87	154.98	135.00	3.15

* A milk-and-cream mixture testing 17.8 per cent fat can easily be standardized by the Pearson-square method.

TABULATION AS A SHORT-CUT FOR ROUTINE CALCULATIONS

In many plants a mix of uniform composition is made from day to day; in addition, the composition of the serum-solids concentrate used does not fluctuate. In that case the amount of serum-solids concentrate required for a given-sized batch will be constant, as also will be the amounts of nonmilk products, no matter how calculated. A table posted in the workroom giving the quantities of these products, along with the total weight and the average fat test of the milk and cream mixture required for the remainder of the standardization, will make unnecessary the repeated calculations for these values.

Table 1 illustrates such a tabulation for specified quantities of a mix to contain 12.0 per cent fat, 10.5 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.35 per cent gelatin, making 37.85 per cent total solids. Here condensed skim milk testing 32.0 per cent milk-solids-not-fat has been used as the serum-solids concentrate. Since the percentages of fat in milk and in cream may be expected to vary considerably from day

TABLE 2

Tabulated Recipes for 100-Pound Batches of Ice-Milk and Ice-Cream Mixes

(Choose any one of the following combinations)

Desired composition of ice-milk or ice-cream mix (15.0 per cent sugar in all cases)	Combination 1					Combination 2				
	Sugar	Gelatin	Skim-milk powder (97.0 per cent total solids)	Milk and cream mixture*		Sugar	Gelatin	Condensed skim-milk (32.0 per cent total solids)	Milk and cream mixture*	
				Weight	Fat test				Weight	Fat test
	pounds	*pounds*	*pounds*	*pounds*	*per cent*	*pounds*	*pounds*	*pounds*	*pounds*	*per cent*
Ice milk:										
4.5 per cent fat and 0.5 per cent gelatin:†										
11.5 per cent milk-solids-not-fat	15.00	0.50	4.89	79.61	5.65	15.00	0.50	18.70	65.80	6.84
13.0 per cent milk-solids-not-fat	15.00	.50	6.59	77.91	5.78	15.00	.50	25.22	59.28	7.59
15.0 per cent milk-solids-not-fat	15.00	.50	8.86	75.64	5.95	15.00	.50	33.91	50.59	8.90
6.0 per cent fat and 0.5 per cent gelatin:†										
11.0 per cent milk-solids-not-fat	15.00	.50	4.47	80.03	7.50	15.00	.50	17.11	67.39	8.90
12.5 per cent milk-solids-not-fat	15.00	.50	6.18	78.32	7.66	15.00	.50	23.63	60.87	9.86
14.0 per cent milk-solids-not-fat	15.00	.50	7.88	76.62	7.83	15.00	.50	30.15	54.35	11.04
Ice cream:										
10.5 per cent fat and 0.4 per cent gelatin:†										
10.5 per cent milk-solids-not-fat	15.00	.40	4.35	80.25	13.08	15.00	.40	16.65	67.95	15.45
11.5 per cent milk-solids-not-fat	15.00	.40	5.49	79.11	13.27	15.00	.40	21.00	63.60	16.51
12.0 per cent fat and 0.4 per cent gelatin:†										
9.5 per cent milk-solids-not-fat	15.00	.40	3.38	81.22	14.77	15.00	.40	12.90	71.70	16.74
10 5 per cent milk-solids-not-fat	15.00	.40	4.51	80.09	14.98	15.00	.40	17.24	67.36	17.81
14.0 per cent fat and 0.3 per cent gelatin:†										
9.0 per cent milk-solids-not-fat	15.00	.30	3.00	81.70	17.14	15.00	.30	11.47	73 23	19.12
10.0 per cent milk-solids-not-fat	15.00	.30	4.14	80.56	17.38	15.00	.30	15.81	68.89	20.32
16.0 per cent fat and 0.3 per cent gelatin:†										
8.0 per cent milk-solids-not-fat	15.00	.30	2.07	82.63	19.36	15.00	.30	7.90	76.80	20.83
9.0 per cent milk-solids-not-fat	15.00	0.30	3.20	81.50	19.63	15.00	0.30	12.25	72.45	22.08

Desired composition of ice-milk or ice-cream mix (15.0 per cent sugar in all cases)	Combination 3					Combination 4				
	Sugar	Gelatin	Condensed milk (8.0 per cent fat, 20.0 per cent milk-solids-not-fat)‡	Milk and cream mixture*		Sugar	Gelatin	Sweetened condensed skim milk (28.0 per cent milk-solids-not-fat, 42.0 per cent sucrose)	Milk and cream mixture*	
				Weight	Fat test				Weight	Fat test
	pounds	*pounds*	*pounds*	*pounds*	*per cent*	*pounds*	*pounds*	*pounds*	*pounds*	*per cent*
Ice milk:										
4.5 per cent fat and 0.5 per cent gelatin:†										
11.5 per cent milk-solids-not-fat	15.00	0.50	36.69	47.81	3.26	7.07	0.50	18.88	73.55	6.12
13.0 per cent milk-solids-not-fat	15.00	.50	49.49	35.01	1.54	4.31	.50	25.46	69.73	6.45
15.0 per cent milk-solids-not-fat	.....	...	.....§	.....	....	0.62	.50	34.24	64.64	6.96
6.0 per cent fat and 0.5 per cent gelatin:†										
11.0 per cent milk-solids-not-fat	15.00	.50	33.58	50.92	6.50	7.75	.50	17.27	74.48	8.06
12.5 per cent milk-solids-not-fat	15.00	.50	46.37	38.13	6.01	4.98	.50	23.86	70.66	8.49
14.0 per cent milk-solids-not-fat	15.00	.50	59.12	25.38	5.00	2.22	.50	30.44	66.84	8.98
Ice cream:										
10.5 per cent fat and 0.4 per cent gelatin:†										
10.5 per cent milk-solids-not-fat	15.00	.40	32.68	51.92	15.20	7.94	.40	16.81	74.85	14.03
11.5 per cent milk-solids-not-fat	15.00	.40	41.22	43.38	16.60	6.10	.40	21.21	72.29	14.52
12.0 per cent fat and 0.4 per cent gelatin:†										
9.5 per cent milk-solids-not-fat	15.00	.40	25.31	59.29	16.82	9.52	.40	13.04	77.04	15.58
10.5 per cent milk-solids-not-fat	15.00	40	33.84	50.76	18.31	7.69	.40	17.41	74.50	16.11
14.0 per cent fat and 0.3 per cent gelatin:†										
9.0 per cent milk-solids-not-fat	15.00	.30	22.50	62.20	19.61	10.13	.30	11.59	77.98	17.95
10.0 per cent milk-solids-not-fat	15.00	.30	31.03	53.67	21.46	8.29	.30	15.97	75.44	18.56
16.0 per cent fat and 0.3 per cent gelatin:†										
8.0 per cent milk-solids-not-fat	15.00	.30	15.50	69.20	21.35	11.65	.30	7.98	80.07	19.98
9.0 per cent milk-solids-not-fat	15.00	0.30	24.04	60.66	23.21	9.80	0.30	12.37	77.53	20.64

* The desired composition of the milk and cream mixture in each case can be secured by using the Pearson-square method of standardization (p. 5).

† The percentage of gelatin used in any one case should depend upon the composition of the mix and the grade of gelatin available. The required gelatin content, with a given grade of gelatin, will decrease with an increase in the fat or total-solids content of the mix. Sodium alginate or possibly other stabilizers may be used in place of gelatin.

‡ The composition of condensed whole milk given here is essentially the same as that required by the federal standard for evaporated milk. By this standard, evaporated milk must contain at least 7.9 per cent milk fat and 18.0 per cent milk-solids-not-fat; but it usually contains slightly in excess of the minimum standards. Evaporated milk sterilized by heat will, if used as a mix ingredient, impart a cooked flavor to ice cream. It may be used satisfactorily, however, without sterilization, and as such is called *condensed milk*.

§ It is not possible to obtain the desired composition if condensed milk is used in this case.

to day, it is usually not practical to tabulate these for specified fat contents. The exact weight of each, however, can easily be calculated by the Pearson-square method each time a mix is made. This is the only calculation required for routine standardization of such mixes. Thus the final step in proportioning the ingredients for the problem illustrated in

TABLE 3

USUAL RELATION OF PERCENTAGE OF FAT TO PERCENTAGE OF OTHER SOLIDS IN MILK*

Milk fat	Milk-solids-not-fat	Total milk solids	Ratio of fat to solids-not-fat	Calculated milk-solids-not-fat in skim-milk portion (serum)
per cent	*per cent*	*per cent*		*per cent*
3.0	8.33	11.33	1 : 2.78	8.59
3.1	8.40	11.50	1 : 2.71	8.67
3.2	8.46	11.66	1 : 2.64	8.74
3.3	8.52	11.82	1 : 2.58	8.81
3.4	8.55	11.95	1 : 2.51	8.85
3.5	8.60	12.10	1 : 2.46	8.91
3.6	8.65	12.25	1 : 2.40	8.97
3.7	8.69	12.39	1 : 2.35	9.02
3.8	8.72	12.52	1 : 2.29	9.06
3.9	8.76	12.66	1 : 2.25	9.12
4.0	8.79	12.79	1 : 2.20	9.16
4.1	8.82	12.92	1 : 2.15	9.20
4.2	8.86	13.06	1 : 2.11	9.25
4.3	8.89	13.19	1 : 2.07	9.29
4.4	8.92	13.32	1 : 2.03	9.33
4.5	8.95	13.45	1 : 1.99	9.37
4.6	8.98	13.58	1 : 1.95	9.41
4.7	9.01	13.71	1 : 1.92	9.45
4.8	9.04	13.84	1 : 1.88	9.50
4.9	9.07	13.97	1 : 1.85	9.54
5.0	9.10	14.10	1 : 1.82	9.58

* The first four columns of this table are taken from: Kelley, E., and C. E. Clements. Market milk, 2d ed. p. 42. John Wiley and Sons, New York, N.Y. 1931. The values given in this table represent averages compiled from more than 250,000 milk samples.

table 1 consists in calculating by this method the exact quantities of milk and of cream with known fat contents to supply the weights of the mixture testing 17.80 per cent fat, which are indicated in column 2 of table 1 for various-sized batches.

The recipes given in table 2 likewise illustrate this method. They are designed to give quantities of ingredients required for 100-pound batches where different serum-solids concentrates are used; also where the composition of the mixes varies.

ALGEBRAIC METHOD

The algebraic method here illustrated depends upon the use of simultaneous equations. The most common type of mix-standardization problem involves three unknowns and three equations, as illustrated below.

TABLE 4

Percentages of Milk-Solids-Not-Fat in Cream Where Its Serum Contains Given Percentages of Milk-Solids-Not-Fat

Fat in cream	Milk-solids-not-fat in cream where the serum contains					
	8.7 per cent	8.8 per cent	8.9 per cent	9.0* per cent	9.1 per cent	9.2 per cent
per cent	*per cent*	*per cent*	*per cent*	*per cent*	*per cent*	*per cent*
15.0	7.40	7.48	7.57	7.65	7.74	7.82
16.0	7.31	7.39	7.48	7.56	7.64	7.73
17.0	7.22	7.30	7.39	7.47	7.55	7.64
18.0	7.13	7.21	7.30	7.38	7.46	7.54
19.0	7.05	7.13	7.21	7.29	7.37	7.45
20.0	6.96	7.04	7.12	7.20	7.28	7.36
21.0	6.87	6.95	7.03	7.11	7.19	7.27
22.0	6.79	6.86	6.94	7.02	7.10	7.18
23.0	6.70	6.78	6.85	6.93	7.01	7.08
24.0	6.61	6.69	6.76	6.84	6.92	6.99
25.0	6.53	6.60	6.68	6.75	6.83	6.90
26.0	6.44	6.51	6.59	6.66	6.73	6.81
27.0	6.35	6.42	6.50	6.57	6.64	6.72
28.0	6.26	6.34	6.41	6.48	6.55	6.62
29.0	6.18	6.25	6.32	6.39	6.46	6.53
30.0	6.09	6.16	6.23	6.30	6.37	6.44
31.0	6.00	6.07	6.14	6.21	6.28	6.35
32.0	5.92	5.98	6.05	6.12	6.19	6.26
33.0	5.83	5.90	5.96	6.03	6.10	6.16
34.0	5.74	5.81	5.87	5.94	6.01	6.07
35.0	5.66	5.72	5.79	5.85	5.92	5.98
36.0	5.57	5.63	5.70	5.76	5.82	5.89
37.0	5.48	5.54	5.61	5.67	5.73	5.80
38.0	5.39	5.46	5.52	5.58	5.64	5.70
39.0	5.31	5.37	5.43	5.49	5.55	5.61
40.0	5.22	5.28	5.34	5.40	5.46	5.52
41.0	5.13	5.19	5.25	5.31	5.37	5.43
42.0	5.05	5.10	5.16	5.22	5.28	5.34
43.0	4.96	5.02	5.07	5.13	5.19	5.24
44.0	4.87	4.93	4.98	5.04	5.10	5.15
45.0	4.79	4.84	4.90	4.95	5.01	5.06
46.0	4.70	4.75	4.81	4.86	4.91	4.97
47.0	4.61	4.66	4.72	4.77	4.82	4.88
48.0	4.52	4.58	4.63	4.68	4.73	4.78
49.0	4.44	4.49	4.54	4.59	4.64	4.69
50.0	4.35	4.40	4.45	4.50	4.55	4.60
55.0	3.92	3.96	4.01	4.05	4.10	4.14
60.0	3.48	3.52	3.56	3.60	3.64	3.68

* A safe average for practical purposes is 9.0 per cent milk-solids-not-fat in the serum of milk and cream. Hence, the values in this column should be used unless it is known that some other value more nearly represents the composition of the sample under consideration.

Problem: Suppose we wish to make 100 pounds of mix to contain 14.0 per cent fat, 9.0 per cent milk-solids-not-fat, 14.5 per cent sugar, 0.5 per cent gelatin, and 0.5 per cent egg-yolk solids. Consider the following ingredients to be available: sugar, gelatin, egg yolk containing 50.0 per cent egg-yolk solids, cream testing 30.0 per cent fat, milk testing

3.6 per cent fat, and condensed whole milk containing 8.0 per cent fat and 20.0 per cent milk-solids-not-fat.

In some cases, milk and cream are analyzed for both fat and total solids; but usually only the percentage of fat is determined. Tables 3 and 4 show the relations between fat and serum solids that are likely to occur, and the values for milk-solids-not-fat given can be used where analytical data are lacking. If some other values are known to represent more nearly the compositions of the products, obviously they should be used.

The amounts of nonmilk products in 100 pounds of mix are found as follows:

$$0.145 \times 100 = 14.5 = \text{pounds of sugar.}$$
$$0.005 \times 100 = 0.5 = \text{pounds of gelatin.}$$
$$\frac{0.005 \times 100}{0.5} = 1.0 = \text{pounds of egg yolk (contains 0.5 pound of egg-yolk solids).}$$

Hence $14.5 + 0.5 + 1.0 = 16.0 =$ pounds of nonmilk products; and $100.0 - 16.0 = 84.0 =$ pounds of milk products.

To solve for the amount of each of the milk products, they will be designated as follows:

$X =$ pounds of 30.0 per cent cream required.
$Y =$ pounds of 3.6 per cent milk required.
$Z =$ pounds of condensed milk required.

Three equations[6] using these three unknowns follow:

(1) Equation for total milk products: $X + Y + Z = 84.0$ pounds.
(2) Equation for fat in mix: $0.30X + 0.036Y + 0.08Z = 14.0$ pounds.
(3) Equation for serum solids in mix: $0.063X + 0.0865Y + 0.20Z = 9.0$ pounds.

One can solve these equations by eliminating one or more unknowns at a time—either by combining two equations by addition or subtraction, or by substituting the value for one unknown in another equation. These are commonly called the addition or subtraction method and the substitution method, respectively. Usually with three unknowns the first method is preferable, although sometimes a combination of the two methods makes the calculations easier. Only the first method will be illustrated here. In this case, one of the equations must be multiplied by a factor such that the coefficients of one of the unknowns in two equations are the same. Thus if equation 3 above is multiplied by 5, the coefficient of Z becomes the same as the coefficient of Z in equation 1. Subtracting:

(1)	$X + Y + Z = 84.00$	
(4)	$0.315X + 0.4325Y + Z = 45.00$	(Equation 3 × 5)
(5)	$0.685X + 0.5675Y = 39.00$	(Equation 1 — equation 4)

[6] If the condensed milk used contains sugar, proper allowance should be made for this in the equations. Thus if it contains 42.0 per cent sucrose, equation 1 above would become $X + Y + 0.58Z = 84.0$.

Similarly, Z can be eliminated from equations 1 and 2 if equation 1 is multiplied by 8, and equation 2 by 100. Subtracting:

(6) $30X + 3.6Y + 8Z = 1{,}400.00$ (Equation 2 × 100)

(7) $8X + 8.0Y + 8Z = 672.00$ (Equation 1 × 8)

(8) $22X - 4.4Y = 728.00$ (Equation 6 — equation 7)

TABULATED CHECK 4

Ingredients used	Weight	Milk fat supplied	Milk-solids-not-fat supplied	Total solids supplied
	pounds	*pounds*	*pounds*	*pounds*
Sugar	14.50	.....	.....	14.50
Gelatin	0.50	.....	.....	0.50
Egg yolk	1.00	.....	.....	0.50
Milk testing 3.6 per cent fat	23.182	0.835	2.005	2.84
Condensed milk testing 8.0 per cent fat, 20.0 per cent milk-solids-not-fat	23.091	1.847	4.618	6.465
Cream testing 30.0 per cent fat	37.727	11.318	2.377	13.695
Totals supplied	100.000	14.000	9.000	38.500
Totals required	100.000	14.000	9.000	38.500

One can eliminate another unknown from equations 5 and 8 by dividing each equation by its respective coefficient of X. Subtracting:

(9) $X + 0.8285Y = 56.934$ (Equation 5 ÷ 0.685)

(10) $X - 0.2000Y = 33.091$ (Equation 8 ÷ 22.0)

(11) $1.0285Y = 23.843$ (Equation 9 — equation 10)

Solving equation 11 for Y,

$$Y = 23.182.$$

Substituting this value of Y in equation 8 and solving for X,

$$22X - (4.4 \times 23.182) = 728.000;$$

$$22X = 830.001;$$

$$X = 37.727.$$

Substituting values for X and Y in equation 1 and solving for Z,

$$37.727 + 23.182 + Z = 84.0;$$

$$Z = 23.091.$$

The results are verified as shown in tabulated check 4.

GRAPHIC METHOD OF STANDARDIZATION

The graphic method of standardizing ice-cream mixes, introduced by J. A. Cross[7] and later reported by W. V. Price[8] as applicable for restandardization as well as for original standardization problems, should prove

[7] Mojonnier, T., and H. C. Troy. Technical control of dairy products. p. 416–18. Mojonnier Brothers Company, Chicago, Ill. 1922.

[8] Price, W. V. A graphical method of proportioning and standardizing ice cream mixes. Journal of Dairy Science 10:292–99. 1927.

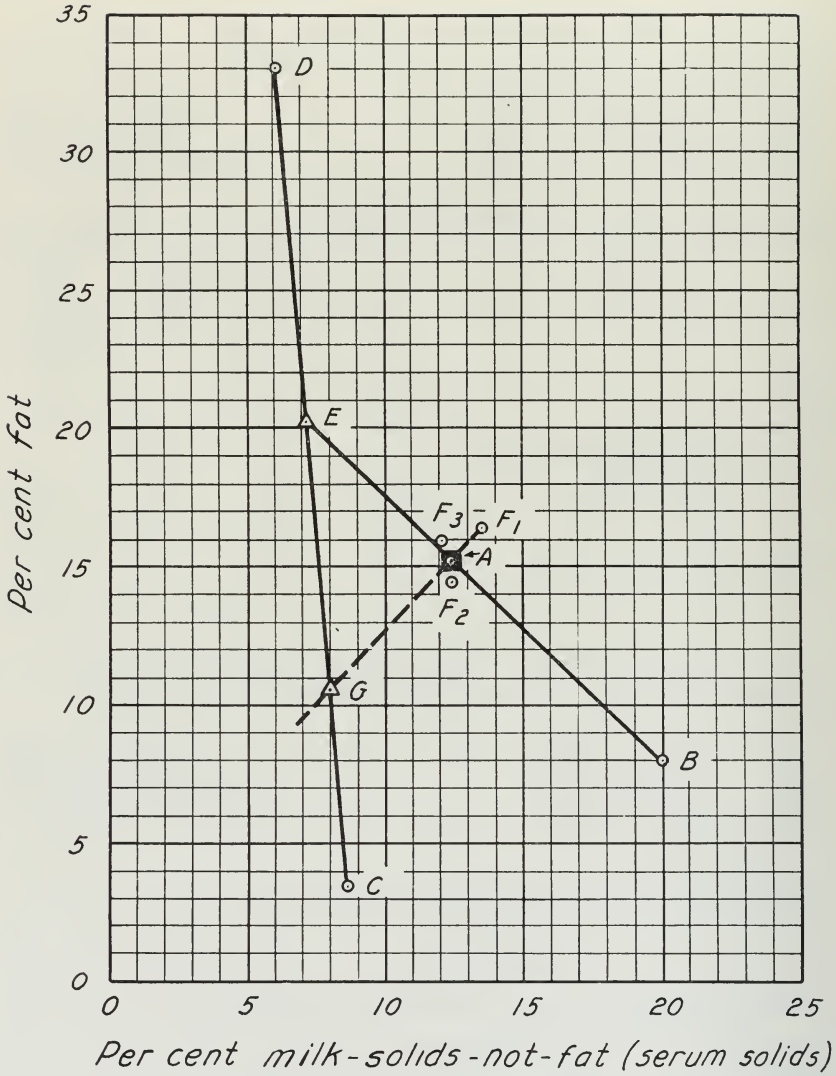


Fig. 1.—Graph for standardizing mixes and restandardizing off batches: *A*, composition of dairy-products portion of desired mix; *B*, composition of condensed milk available; *C*, composition of milk available; *D*, composition of cream available; *E* and *G*, composition of two milk and cream mixtures determined graphically; F_1, F_2, and F_3, compositions of the dairy-products portions of three off batches.

satisfactory for those accustomed to graphs. The method is simple, and the calculations are relatively few. Inaccuracies introduced by plotting and reading the graph need not be great enough to make the method impracticable. In fact, its simplicity and especially its adaptability for restandardizing off batches recommend its more general use.

Problem: The method can perhaps best be explained by an illustration. Suppose we wish to make 100 pounds of mix to test 13.0 per cent fat, 10.5 per cent milk-solids-not-fat, 14.6 per cent sugar, and 0.4 per cent gelatin; and the products available are sugar, gelatin, cream testing 33.0 per cent fat, milk testing 3.6 per cent fat, and condensed milk testing 8.0 per cent fat and 20.0 per cent milk-solids-not-fat.

The calculations for the nonmilk products are the same as in any of the other methods previously considered. Thus

$$0.146 \times 100 = 14.6 = \text{pounds of sugar; and}$$
$$0.004 \times 100 = 0.4 = \text{pound of gelatin.}$$
$$\text{Hence } 14.6 + 0.4 = 15.0 = \text{pounds of nonmilk products; and}$$
$$100.0 - 15.0 = 85.0 = \text{pounds of milk products.}$$

To solve the problem by this method, first determine the percentage of fat and the percentage of milk-solids-not-fat in the milk products. Thus

$$\frac{13.0}{85} \times 100 = 15.29 = \text{per cent fat in the milk-products portion of the mix; and}$$

$$\frac{10.5}{85} \times 100 = 12.35 = \text{per cent milk-solids-not-fat in the milk-products portion of the mix.}$$

Using coördinate paper as indicated in figure 1, let the horizontal axis represent the percentage of milk-solids-not-fat, and let the vertical axis represent the percentage of fat. Now, on this graph, plot point *A*, using the values just calculated to represent the composition of the milk-products portion of the mix to be standardized. Besides using the data given in the problem, refer to tables 3 and 4 for the percentages of milk-solids-not-fat in the milk and cream; then using these values, plot points representing the composition of the three milk products available for standardization: *B* representing the condensed milk, *C* the milk, and *D* the cream.

Obviously the line segment *CD* represents the composition of all possible mixtures of milk and cream represented by the points *C* and *D*. Now the line *BA*, when extended, will intersect the line *CD* at *E*, representing the composition of the milk and cream mixture, which, when combined in the correct proportion with the condensed milk of *B* composition, will result in a mixture of *A* composition.[9] The other designated points in figure 1 can be ignored for the present, but will be used later in a problem of restandardization presented on page 36.

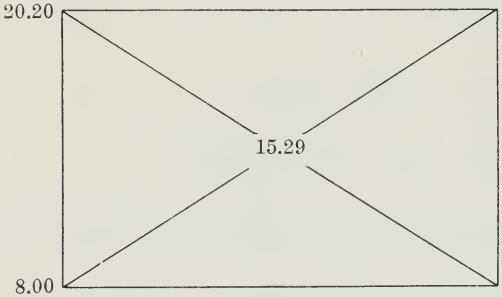
The problem can be finished in either of two ways:

Procedure 1: Using the Pearson square, standardize in each case for

[9] When used for routine standardization, the work can be simplified if a large sheet of coördinate paper is used and the necessary points are represented by pins. Lines can be indicated by thread stretched between the pins.

either the milk fat or the milk-solids-not-fat. Usually it is preferable to standardize on the basis of the fat as illustrated in the problem below.

Determine the pounds of condensed milk by considering the fact that condensed milk of *B* composition contains 8.0 per cent fat and the milk and cream mixture of *E* composition contains 20.2 per cent fat, as can be seen by referring to the graph. Thus



7.29 = unit parts of milk and cream mixture having *E* composition

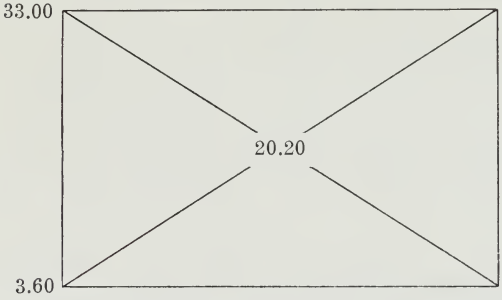
4.91 = unit parts of condensed milk having *B* composition

Hence $7.29 + 4.91 = 12.20$ = total unit parts of mixture containing 15.29 per cent fat;

$$\text{and the pounds of condensed milk} = \frac{4.91}{12.20} \times 85.0 = 34.21.$$

Therefore the pounds of milk and cream mixture = 85.00 — 34.21 = 50.79.

Next determine the pounds of milk and cream required to make 50.79 pounds of the mixture containing 20.20 per cent fat. Thus



16.60 = unit parts of cream

12.80 = unit parts of milk

Hence $16.60 + 12.80 = 29.40$ = total unit parts of milk and cream mixture;

$$\text{and the pounds of cream} = \frac{16.60}{29.40} \times 50.79 = 28.68.$$

Therefore the pounds of milk = 50.79 — 28.68 = 22.11.

Procedure 2: The lengths of line segments on the graph can be used to determine the proportion of the ingredients required. The unit of length chosen is immaterial as long as it is uniform throughout the calculations, but usually the most convenient for this purpose is the unit of space on the graph. One can obtain the length in any case by using dividers and reading the length off the base line or by simply subtracting

on one of the scales the difference between two points; for example, length of line segment $CD = 33.00 - 3.60 = 29.40$; length of line segment $AB = 15.29 - 8.00 = 7.29$. By this last method of determining the length, the pounds of each ingredient can be calculated as follows:

$$\text{Pounds of condensed milk} = \frac{\text{length of line } EA}{\text{length of line } EB} \times \text{pounds of milk products; thus}$$

$$\frac{4.91}{12.20} \times 85.0 = 34.21 = \text{pounds of condensed milk.}$$

$$\text{Pounds of milk and cream mixture} = 85.00 - 34.21 = 50.79.$$

$$\text{Pounds of cream} = \frac{\text{length of line } CE}{\text{length of line } CD} \times \text{pounds of milk and cream;}$$

$$\frac{16.60}{29.40} \times 50.79 = 28.68 = \text{pounds of cream.}$$

$$\text{Pounds of milk} = 50.79 - 28.68 = 22.11.$$

TABULATED CHECK 5

Ingredients used	Weight	Milk fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Sugar	14.60	....	....	14.60
Gelatin	0.40	....	....	0.40
Condensed milk testing 8.0 per cent fat and 20.0 per cent milk-solids-not-fat	34.21	2.74	6.84	9.58
Cream testing 33.0 per cent fat	28.68	9.46	1.73	11.19
Milk testing 3.6 per cent fat	22.11	0.80	1.91	2.71
Totals supplied	100.00	13.00	10.48	38.48
Totals required	100.00	13.00	10.50	38.50

In this method of determining the length of the line segment, the calculations are identical with those by the Pearson-square method except that ordinarily the measurements cannot be scaled so accurately as they can be computed.

Tabulated check 5 shows that the calculations are correct for practical purposes.

STANDARDIZING MIXES TO BE MADE IN A VACUUM PAN

The calculations for standardizing mixes to be prepared in a vacuum pan are fewer than in the more usual process where a serum-solids concentrate is used. The weights of nonmilk products are calculated for a definite finished weight of mix. Then the composition of a mixture of milk and cream having the correct ratio of $\frac{\text{fat}}{\text{milk-solids-not-fat}}$ is calculated, or determined by reference to a table such as table 5 or a graph such as figure 2, and finally the weight of this mixture required is calcu-

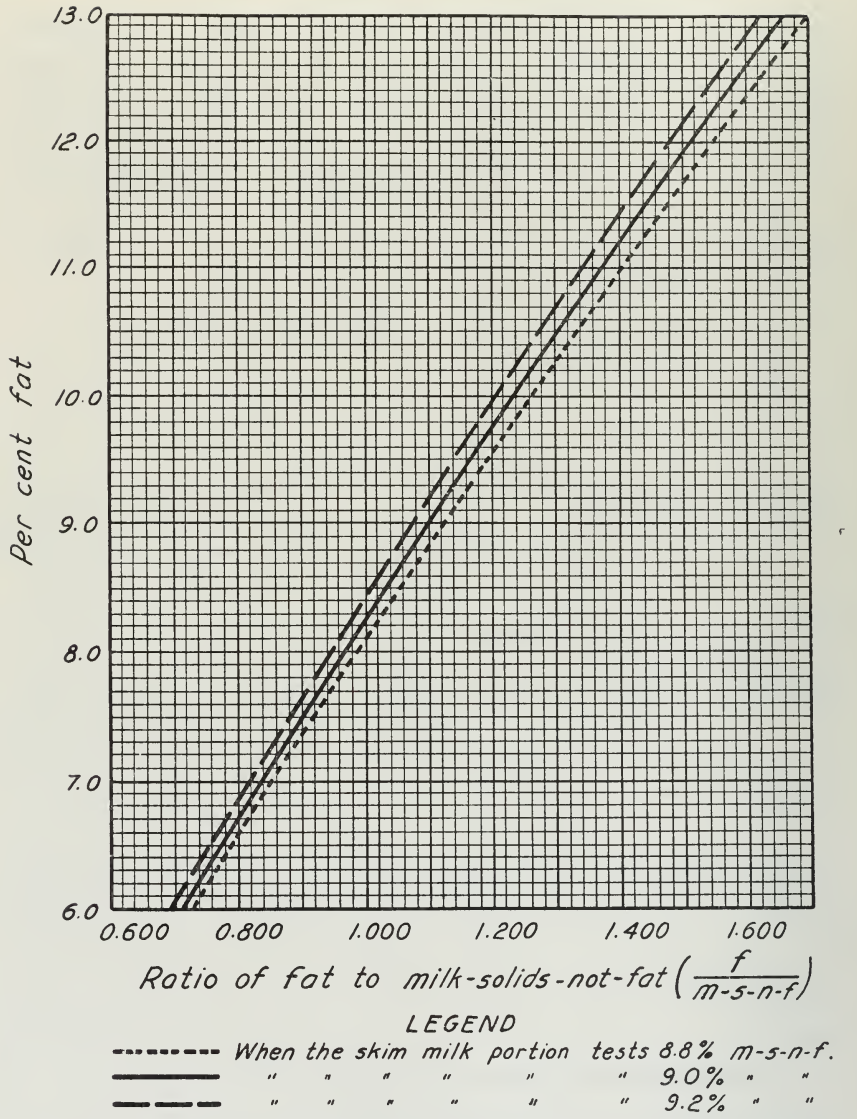


Fig. 2.—The percentage of fat in milk and cream mixtures as related to the ratio of fat to milk-solids-not-fat $\left[\frac{\text{fat}}{\text{milk-solids-not-fat}}\right]$ in such mixtures.

lated by the Pearson-square method. The following example will illustrate this procedure.

Problem: Suppose we wish to determine the amounts of ingredients required to prepare 1,000 pounds of mix, the latter to be made in a

vacuum pan. The desired composition of the mix is 14.0 per cent fat, 10.0 per cent milk-solids-not-fat, 14.7 per cent sugar, and 0.3 per cent gelatin. Milk testing 3.7 per cent fat and cream testing 40.0 per cent fat are available in addition to sugar and gelatin.

Although the order in which these ingredients are determined is of no consequence, for convenience the calculations are listed in steps.

Calculate the amounts of nonmilk products required:

$0.147 \times 1{,}000 = 147.0 =$ pounds of sugar; and
$0.003 \times 1{,}000 = 3.0 =$ pounds of gelatin.

Calculate the ratio of $\frac{\text{fat}}{\text{milk-solids-not-fat}}$ desired in the final mix. Thus $\frac{14.0}{10.0} = 1.40$.

By referring to either table 5 or figure 2, determine the percentage of fat in the mixture of milk and cream having the ratio 1.40. This amount proves to be 11.2 per cent fat.

It may also be calculated in the following manner:

Let $F =$ fat fraction of the desired mix,
$S =$ milk-solids-not-fat fraction of the desired mix,
$A =$ milk-solids-not-fat fraction of the serum (skim-milk portion) of the mixture of milk and cream,
$X =$ fat fraction of the desired milk and cream mixture.

Hence $(1 - X)A =$ milk-solids-not-fat fraction of the milk and cream mixture; and

$$\frac{F}{S} = \frac{X}{(1-X)A}.$$

Solving this equation for X,

$$X = \frac{AF}{S + AF}.$$

Assuming that the serum tests 9.0 per cent milk-solids-not-fat, the value of A becomes 0.09. When substituted in the preceding equation, it is as follows:

$$X = \frac{0.09F}{S + 0.09F}.$$

If we let $R = \frac{F}{S} =$ ratio of milk fat to milk-solids-not-fat in the desired mix, then the equation above becomes

$$X = \frac{0.09R}{1 + 0.09R}.$$

Either equation will give the same results provided the correct values are substituted in each case.

TABLE 5

RATIOS OF FAT TO MILK-SOLIDS-NOT-FAT IN MIXTURES OF MILK AND CREAM*

Per cent fat in mixture	Ratio of fat to milk-solids-not-fat in *mixture* with the indicated percentages of milk-solids-not-fat in *serum* of mixture					
	With 8.7 per cent	With 8.8 per cent	With 8.9 per cent	With 9.0† per cent	With 9.1 per cent	With 9.2 per cent
6.0	0.734*	0.726	0.717	0.709	0.702	0.694
6.1	0.747	0.738	0.730	0 722	0.714	0.706
6.2	0.760	0.752	0.743	0.735	0.726	0.718
6.3	0.773	0.765	0.755	0.747	0.739	0.731
6.4	0.786	0.777	0.768	0.760	0.751	0.743
6.5	0.800	0.790	0.781	0.772	0.764	0.756
6.6	0.812	0.803	0.794	0.785	0.776	0.768
6.7	0.825	0.816	0.807	0.798	0.789	0.781
6.8	0.838	0.829	0.820	0.810	0.802	0.793
6.9	0.852	0.842	0.832	0.823	0.815	0.805
7.0	0.865	0.856	0.845	0.836	0.827	0.818
7.1	0.879	0.868	0.859	0.849	0.840	0.830
7.2	0.892	0.881	0.872	0.862	0.853	0.843
7.3	0.906	0.895	0.885	0.875	0.865	0.856
7.4	0.918	0.908	0.898	0.888	0.878	0.869
7.5	0.932	0.921	0.911	0.900	0.891	0.881
7.6	0 945	0.935	0.925	0.913	0.904	0.894
7.7	0.959	0.948	0.938	0.927	0.917	0.907
7.8	0.973	0.962	0.950	0.940	0.930	0.920
7.9	0.986	0.975	0.963	0.953	0.943	0.933
8.0	1.000	0.988	0.977	0.966	0.956	0.946
8.1	1.013	1.001	0.990	0.979	0.969	0.959
8.2	1.026	1.015	1.004	0.993	0.982	0.970
8.3	1.040	1.029	1.017	1.006	0.995	0.983
8.4	1.054	1.042	1.031	1.019	1.007	0.996
8.5	1.068	1.056	1.044	1.032	1.020	1.010
8.6	1.082	1.070	1.058	1.045	1.034	1.023
8.7	1.096	1.083	1.070	1.058	1.047	1.036
8.8	1.110	1.096	1.084	1.072	1.060	1.049
8.9	1.122	1.110	1.097	1.085	1.074	1.062
9.0	1.136	1.124	1.111	1.099	1.087	1.075
9.1	1.150	1.138	1.125	1.112	1.100	1.089
9.2	1.165	1.151	1.139	1.126	1.114	1.102
9.3	1.179	1.165	1.152	1.140	1.127	1.115
9.4	1.193	1.179	1.166	1.153	1.141	1.127
9.5	1.207	1.193	1.180	1.166	1.153	1.140
9.6	1.221	1.206	1.193	1.179	1.166	1.154
9.7	1.234	1.220	1.206	1.193	1.180	1.167
9.8	1.248	1.234	1.220	1.207	1.194	1.181
9.9	1.263	1.248	1.234	1.221	1.207	1.194
10.0	1.277	1.263	1.248	1.235	1.221	1.208
10.1	1.292	1.277	1.263	1.248	1.235	1.221
10.2	1.306	1.291	1.277	1.262	1.248	1.235
10.3	1.321	1.305	1.291	1.276	1.262	1.248
10.4	1.333	1.320	1.305	1.290	1.276	1.262
10.5	1.348	1.332	1 317	1.303	1.290	1.276

* The ratios in this table were derived by dividing the percentage of fat by the percentage of milk-solids-not-fat for each mixture. Thus the first value in the table is obtained as follows: A mixture containing 6 per cent fat will contain 94 per cent serum. As indicated in the column heading, the serum contains 8.7 per cent milk-solids-not-fat. Hence the mixture contains (0.94×8.7), or 8.178 per cent milk-solids-not-fat. The ratio would then be $\frac{6.0}{8.178}=0.734$.

† Values given in this column should be used unless, in a specific case, a value given in one of the other columns more nearly represents the relation which exists between the fat and milk-solids-not-fat.

TABLE 5—(*Continued*)

Per cent fat in mixture	Ratio of fat to milk-solids-not-fat in *mixture* with the indicated percentages of milk-solids-not-fat in *serum* of mixture					
	With 8.7 per cent	With 8.8 per cent	With 8.9 per cent	With 9.0† per cent	With 9.1 per cent	With 9.2 per cent
10.6	1.362	1.347	1.332	1.317	1.302	1.290
10.7	1.377	1.361	1.346	1.331	1.316	1.302
10.8	1.392	1.376	1.360	1.345	1.330	1.315
10.9	1.406	1.390	1.375	1.359	1.344	1.329
11.0	1.421	1.405	1.389	1.373	1.358	1.343
11.1	1.436	1.419	1.403	1 388	1.372	1.357
11.2	1.449	1.434	1.418	1.402	1.386	1.371
11.3	1.464	1.447	1.432	1.416	1.400	1.385
11.4	1.479	1.462	1.445	1.430	1.414	1.399
11.5	1.494	1.476	1.459	1.443	1.429	1.413
11.6	1.508	1.491	1.474	1.457	1.443	1.427
11.7	1.523	1.506	1.489	1.472	1.455	1.441
11.8	1.538	1.521	1.503	1.486	1.469	1.455
11.9	1.554	1.535	1.518	1.501	1.484	1.467
12.0	1.567	1.550	1.533	1.515	1.498	1.481
12.1	1.582	1.563	1.547	1.530	1.513	1.496
12.2	1.597	1.578	1.562	1.544	1.527	1.510
12.3	1.612	1.593	1.575	1.559	1.541	1.524
12.4	1.627	1.608	1.590	1.574	1.556	1.538
12.5	1.643	1.623	1.605	1.586	1.570	1.553
12.6	1.658	1.638	1.620	1.601	1.585	1.567
12.7	1.671	1.654	1.634	1.616	1.599	1.582
12.8	1.686	1.669	1.649	1.631	1.612	1.596
12.9	1.702	1.684	1.665	1.645	1.627	1.610
13.0	1.717	1.697	1.680	1.660	1.641	1.625

† Values given in this column should be used unless, in a specific case, a value given in one of the other columns more nearly represents the relation which exists between the fat and milk-solids-not-fat.

TABULATED CHECK 6

Ingredients used	Weight	Milk fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Sugar	147.0	.....	.....	147.00
Gelatin	3.0	.....	.....	3.00
Cream testing 40.0 per cent fat	258.3	103.32	13.95	117.27
Milk testing 3.7 per cent fat	991.7	36.69	86.18	122.87
Totals supplied	1,400.0	140.01	100.13	390.14
Totals required	1,000.0	140.00	100.00	390.00

In this problem $F = 0.14$ and $S = 0.10$. Hence, using $X = \frac{0.09F}{S + 0.09F}$,

$$X = \frac{0.09 \times 0.14}{0.10 + (0.09 \times 0.14)} = 0.1119.$$

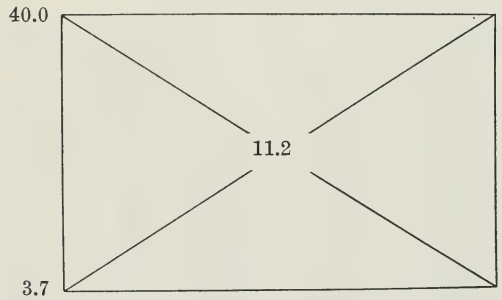
This equals 11.19 per cent or, for practical purposes, 11.2 per cent.

Then calculating the pounds of this milk and cream mixture required:

$0.14 \times 1{,}000 = 140.0 =$ pounds of fat required for the finished mix, or

$\frac{140.00}{0.112} = 1{,}250 =$ pounds of mixture testing 11.2 per cent fat required.

Finally the pounds of milk and cream, respectively, required to prepare this mixture above calculated by the Pearson-square method. Thus



7.5 = unit parts of 40 per cent cream

28.8 = unit parts of 3.7 per cent milk

7.5 + 28.8 = 36.3 = the total unit parts of the mixture testing 11.2 per cent fat.

Hence the pounds of cream testing 40.0 per cent fat $= \frac{7.5}{36.3} \times 1{,}250 = 258.3$,

and the pounds of milk testing 3.7 per cent fat = 1,250.0 — 258.3 = 991.7.

These calculations are verified in tabulated check 6.

As shown in tabulated check 6, the required solids for 1,000 pounds of mix of the desired composition are supplied by a weight of 1,400 pounds. Hence all that remains to be done is to evaporate the amount of water represented by the difference between that desired and the mixture above; that is, 1,400 — 1,000 = 400 = pounds of water to be evaporated. The correct composition is determined by using a hydrometer that gives the specific gravity corresponding to a given total-solids content at the temperature of operation.

Calculating Specific Gravity and Baumé Reading of Mix in Vacuum Pan.—Sommer[10] gives the following method of calculating the specific gravity of ice-cream mixes:

$$\left.\begin{matrix}\text{Specific}\\ \text{gravity}\\ \text{of mix}\\ \text{at } 60^\circ \text{ F}\end{matrix}\right\} = \frac{100}{\frac{\text{per cent fat in mix}}{0.93} + \frac{\text{per cent (sugar + gelatin + milk-solids-not-fat) in mix}}{1.58} + \text{per cent water in mix.}}$$

By this formula, the mix just illustrated would have a specific gravity as follows:

$$\text{Specific gravity at } 60^\circ \text{ F} = \frac{100}{\frac{14.0}{0.93} + \frac{(14.7 + 0.3 + 10.0)}{1.58} + 61.0} = 1.0884.$$

Although a specific-gravity hydrometer may be used, more commonly a Baumé hydrometer is employed to ascertain the desired degree of

[10] Sommer, H. H. Theory and practice of ice cream making. 3d ed., p. 146. Published by the author, Madison, Wisconsin. 1938.

concentration of the mix in the vacuum pan. The Baumé hydrometer reading can be readily calculated if the specific gravity is known. Thus

$$\text{Degrees Baumé} = 145 - \frac{145}{\text{specific gravity}}.$$

$$\text{Substituting, we obtain degrees Baumé} = 145 - \frac{145}{1.0884} = 11.78.$$

Since the Baumé reading will be taken at approximately 120° F, whereas the calculated values according to the method just given apply to 60° F, it is necessary to make a correction. If 0.025 degrees Baumé is subtracted for every degree above 60° F, at 120° F the correction would be 1.5 degrees Baumé, and the corrected reading in degrees Baumé would be 11.78 — 1.5 = 10.28. The exact correction to use will depend not only upon the temperature but upon the composition of the mix.

Admittedly, the method of calculating specific gravity and the correction of Baumé reading for temperature just illustrated are only approximations. Slight variations from these results can be expected in any particular case. If the vacuum-pan method of processing mixes is adopted, samples of mix with the desired composition should be heated to the temperatures at which readings are to be taken regularly, and the correct readings at these temperatures determined. By this procedure the operator should duplicate results without difficulty. In actual practice, for safety, the mix is often slightly overcondensed; then, after analysis, sufficient water is added to adjust it to the desired composition.

USING MISCELLANEOUS INGREDIENTS IN STANDARDIZING MIXES

Occasionally in some ice-cream plants and oftener in others, it is desired to use the entire available supply of several ingredients, which by themselves do not supply the required amount of mix components, and to use sufficient of other available ingredients to complete the standardization. In such a case there may be four or more dairy products used, whereas, in most of the illustrations given thus far, only three dairy products were considered available in quantities sufficient to supply the milk solids in the mix being standardized. Let us illustrate this situation by a specific case.

Problem: Suppose we wish to prepare 2,400 pounds of ice-cream mix to contain 15.0 per cent milk fat, 9.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.3 per cent gelatin. Assume that we wish to use all the following products: 160 pounds of cream testing 41.5 per cent fat, 50 pounds of cream testing 25.0 per cent fat, and 200 pounds of milk testing 3.8 per cent fat. The remainder of the mix is to be prepared from sugar, gelatin, cream testing 40.0 per cent fat, milk testing 3.7 per cent fat, and condensed whole milk testing 8.0 per cent fat and 18.5 per cent milk-

solids-not-fat. Assume that the supply of each of these products is adequate for our purpose.

The first step in a case of this kind is to make a tabulation showing what will be added by the products which are to be completely utilized.

TABULATED CHECK 7

Ingredients used	Weight	Fat supplied	Milk-solids-not-fat supplied	Total solids supplied
	pounds	*pounds*	*pounds*	*pounds*
Cream testing 41.5 per cent fat	160.0	66.40	8.42	74.82
Cream testing 25.0 per cent fat	50.0	12.50	3.38	15.88
Milk testing 3.8 per cent fat	200.0	7.60	17.32	24.92
Totals supplied	410.0	86.50	29.12	115.62
Totals required	2,400.0	360.00	216.00	943.20
Difference to be supplied	1,990.0	273.50	186.88	827.58

TABULATED CHECK 8

Ingredients used	Weight	Fat supplied	Milk-solids-not-fat supplied	Total solids supplied
	pounds	*pounds*	*pounds*	*pounds*
Cream testing 41.5 per cent fat	160.00	66.40	8.42	74.82
Cream testing 25.0 per cent fat	50.00	12.50	3.38	15.88
Milk testing 3.8 per cent fat	200.00	7.60	17.32	24.92
Condensed milk testing 8.0 per cent fat and 18.5 per cent milk-solids-not-fat	640.34	51.23	118.46	169.69
Cream testing 40.0 per cent fat	512.18	204.87	27.66	232.53
Milk testing 3.7 per cent fat	470.28	17.40	40.76	58.16
Sugar	360.00	.....	.....	360.00
Gelatin	7.20	.....	.....	7.20
Totals supplied	2,400.00	360.00	216.00	943.20
Totals required	2,400.00	360.00	216.00	943.20

By subtracting these values from the totals desired we arrived at values yet to be supplied. This is illustrated in tabulated check 7.

The remainder of the calculations are the same as that for standardizing mixes using three dairy products. We can proceed by any one of several methods. If we use the serum-point or formula method, the calculation is as follows:

The rest of the mix to be prepared equals 1,990 pounds. It must contain 360 pounds of sugar and 7.2 pounds of gelatin (an amount sufficient for 2,400 pounds of mix), 273.5 pounds of milk fat, and 186.88 pounds of milk-solids-not-fat, making a total of 827.58 pounds of total solids.

This is equivalent to 367.2 pounds of nonmilk products and 1,622.8 pounds of milk products. The latter must contain 273.5 pounds of fat and 1,349.3 pounds of mix serum, which in turn must contain 186.88 pounds of milk-solids-not-fat. The calculations follow:

$$\text{Pounds of condensed whole milk} = \frac{186.88 - (0.09 \times 1{,}349.3)}{0.185 - (0.09 \times 0.92)} = 640.34.$$

$$\text{Pounds of cream} = \frac{273.50 - 51.23 - (0.037 \times 982.46)}{0.400 - 0.037} = 512.18.$$

Pounds of milk $= 982.46 - 512.18 = 470.28.$

Pounds of sugar $= 2{,}400 \times 0.15 = 360.$

Pounds of gelatin $= 2{,}400 \times 0.003 = 7.2.$

As can be seen from tabulated check 8, these calculations give the correct combination of ingredients for the desired composition.

MAKING A MAXIMUM AMOUNT OF MIX FROM A LIMITED SUPPLY OF CERTAIN INGREDIENTS

Sometimes one may wish to make the maximum amount of mix, of a given composition, from a limited supply of certain ingredients. Although the milk-solids-not-fat concentrate may be the limiting factor in some cases, the cream, or fat concentrate, usually determines the maximum amount of mix made. Since, in either case, the basis of solution depends upon the same principle, only one example will be given, in which cream is considered the limiting factor.

If cream is the only source of fat, which in turn is the limiting factor, the maximum weight of mix can be calculated as follows:

$$\frac{\text{Pounds of fat in available cream}}{\text{Fraction of fat in desired mix}} = \text{maximum weight in pounds of desired mix.}$$

Thus if the available cream contains 180 pounds of fat and the desired mix must contain 12 per cent fat, then

$$\frac{180}{0.12} = 1{,}500 = \text{maximum pounds of desired mix.}$$

The standardization can now be completed by any one of the methods already illustrated, but it seems unnecessary to continue the calculations here.

The following illustration considers a slightly different type of problem even though cream is still the limiting factor.

Problem: Suppose the following products are available: 180 pounds of cream testing 36 per cent fat, 440 pounds of milk testing 3.7 per cent fat, 1,000 pounds of skim-milk powder testing 97 per cent total solids, 2,000 pounds of sugar, 100 pounds of gelatin, and 150 pounds of egg yolk testing 50 per cent egg-yolk solids. Assume that we desire to make

the maximum weight of mix to contain 14.0 per cent fat, 9.5 per cent milk-solids-not-fat, 15.0 per cent sugar, 0.5 per cent egg-yolk solids, and 0.3 per cent gelatin.

In such a problem we must know the proportion of ingredients available that will make a definite quantity of mix with the desired compo-

TABULATED CHECK 9

Ingredients used	Weight	Fat supplied	Milk-solids-not-fat supplied	Total solids supplied
	pounds	*pounds*	*pounds*	*pounds*
Skim-milk powder testing 97 per cent total solids	3.67	.....	3.56	3.56
Cream testing 36.0 per cent fat	34.18	12.30	1.97	14.27
Milk testing 3.7 per cent fat	45.85	1.70	3.97	5.67
Sugar	15.00	.....	.....	15.00
Egg yolk	1.00	.....	.....	0.50
Gelatin	0.30	.....	.....	0.30
Totals supplied	100.00	14.00	9.50	39.30
Totals required	100.00	14.00	9.50	39.30

TABULATED CHECK 10

Ingredients used	Weight	Fat supplied	Milk-solids-not-fat supplied	Total solids supplied
	pounds	*pounds*	*pounds*	*pounds*
Skim-milk powder testing 97 per cent total solids	19.33	.....	18.75	18.75
Cream testing 36.0 per cent fat	180.00	64.80	10.37	75.17
Milk testing 3.7 per cent fat	241.46	8.93	20.93	29.86
Sugar	78.99	.....	.....	78.99
Egg yolk	5.27	.....	.....	2.63
Gelatin	1.58	.....	.....	1.58
Totals supplied	526.63	73.73	50.05	206.98
Totals required	526.62	73.73	50.03	206.96

sition. It is more convenient to have these data for 100 pounds of mix, though the data for any other quantity can be used. Tabulated check 9 gives the information desired for 100 pounds of mix.

As inspection reveals, cream is the limiting factor as far as the supply of ingredients is concerned. Cream represents, furthermore, 34.18 per cent of the weight of the total mix; that is, 34.18 pounds of cream are required for every 100 pounds of mix. Therefore

$$\frac{180}{34.18} \times 100 = 526.62 = \text{maximum weight in pounds of desired mix.}$$

Since 526.62 pounds of mix can be made, this amount can be standardized by any one of the methods previously illustrated for this purpose. In this case it seems simpler to take into consideration the percentage weight of each ingredient of the mix shown in tabulated check 9 as a basis of the calculation. Thus:

Pounds of skim-milk powder = 526.62 × 0.0367 = 19.33.
Pounds of cream = 526.62 × 0.3418 = 180.00.
Pounds of milk = 526.62 × 0.4585 = 241.46.
Pounds of sugar = 526.62 × 0.15 = 78.99.
Pounds of egg yolk = 526.62 × 0.01 = 5.27.
Pounds of gelatin = 526.62 × 0.003 = 1.58.

Tabulated check 10 shows these calculations to be correct for all practical purposes.

CORRECTING OFF BATCHES BY RESTANDARDIZATION

Often an ice-cream mix does not have the desired composition. If it deviates appreciably from the standard adopted, it should be restandardized—that is, adjusted to the desired values. Slight variations from the desired composition ordinarily need not be corrected unless they bring the mix below the legal standard.

The composition of the mix is usually determined by analyzing it for fat and total solids, then assuming that the nonmilk products are present in the correct proportion and computing the milk-solids-not-fat by difference. To illustrate: suppose a mix contains 13.0 per cent fat and 39.5 per cent total solids, whereas it would have contained 12.5 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.5 per cent gelatin, if no mistake had occurred in the original standardization or processing. The sugar and gelatin contents are assumed to be correct, since, generally speaking, errors are less likely to occur in the sugar and gelatin contents of the mix than elsewhere. Hence the percentages of fat in the off batch and the desired percentages of sugar and gelatin will all be subtracted from the total solids in the off batch, as a basis of calculating the milk-solids-not-fat contained in the batch under consideration. Thus

39.5 — (13.0 + 15.0 + 0.5) = 11.0 = per cent of milk-solids-not-fat in the off batch.

One can restandardize a mix by adding the minimum weight of ingredients to correct the off batch or by simply adding a convenient weight of ingredients, provided the latter is large enough to permit the desired correction in composition. In some cases there is a decided advantage in a minimum-sized final batch; in others there is little or no choice between the two methods of deciding the weight of the final mix except for convenience in calculating.

RESTANDARDIZING ON AN ARBITRARY-WEIGHT BASIS

If the mix is restandardized by the use of an arbitrary quantity equaling or exceeding the minimum required, almost any method used for the original standardization will serve. In such a case the off batch is considered one of the ingredients that must be completely utilized. The remainder of the mix can be adjusted, and calculations for its standardization made, so that when the two portions are combined the final mixture will have the desired composition.

The following problem will illustrate restandardization where the mix is increased by an arbitrary quantity.

TABULATED CHECK 11

	Fat	Milk-solids-not-fat	Sugar	Gelatin	Total solids
	pounds	*pounds*	*pounds*	*pounds*	*pounds*
Amount required for 130 pounds corrected mix	18.20	13.00	19.50	0.65	51.35
Amount supplied by 100 pounds of the off batch	12.50	11.00	15.00	0.50	39.00
Difference to be supplied in 30 pounds added for restandardization	5.70	2.00	4.50	0.15	12.35

Problem: Restandardize 100 pounds of mix testing 12.5 per cent fat and 39.0 per cent total solids so that the final mixture will contain 14.0 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.5 per cent gelatin. The milk products available are cream testing 30.0 per cent fat, milk testing 3.6 per cent fat, and condensed skim milk testing 32.0 per cent milk-solids-not-fat. The nonmilk products, sugar and gelatin, are also available.

Arbitrarily assume that a total of 30 pounds will be added to correct the off batch.

Tabulated check 11 summarizes the problem.

Any one of four methods—Pearson square, serum-point, algebraic, and graphic—illustrated in the section devoted to standardization can be used in solving this problem. Only the solution by the serum-point method will be given here.

As tabulated check 11 shows, the correcting batch of 30.0 pounds must contain 4.65 pounds of nonmilk products and 25.35 pounds of milk products. The fat present in these added milk products amounts to 5.70 pounds; consequently, they will contain 19.65 pounds of serum, which is the difference between the weight of milk products and the fat contained therein.

Proceed by substituting the correct values in the serum-point formulas given on page 10:

$$\text{Pounds of condensed skim milk required} = \frac{2.00 - (0.09 \times 19.65)}{0.32 - 0.09} = 1.01.$$

$$\text{Pounds of milk and cream} = 25.35 - 1.01 = 24.34.$$

$$\text{Pounds of cream} = \frac{5.70 - (0.036 \times 24.34)}{0.30 - 0.036} = 18.27.$$

$$\text{Pounds of milk} = 24.34 - 18.27 = 6.07.$$

Tabulated check 12 verifies these calculations.

TABULATED CHECK 12

Ingredients used	Weight	Fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Off batch	100.00	12.50	11.00	39.00
Sugar	4.50	.....	.....	4.50
Gelatin	0.15	.....	.....	0.15
Condensed skim milk testing 32.0 per cent serum solids	1.01	.....	0.32	0.32
Cream testing 30.0 per cent fat	18.27	5.48	1.15	6.63
Milk testing 3.6 per cent fat	6.07	0.22	0.53	0.75
Totals supplied	130.00	18.20	13.00	51.35
Totals required	130.00	18.20	13.00	51.35

Where this method is employed, the procedure is the same whether the milk fat or milk-solids-not-fat in the off batch are both too high, or both too low, or whether one is too high and the other too low. The only prerequisite is that in any case the correcting batch equal or exceed the minimum weight required for restandardization.

RESTANDARDIZING ON A MINIMUM-WEIGHT BASIS

Ordinarily, though not always, it is an advantage to correct an off batch by increasing the weight as little as possible. Whether or not this fact makes the problem more difficult will depend upon the correction to be made and upon the method employed for restandardization.

By the graphic method the worker can restandardize on the basis of a minimum weight more easily than on the basis of any larger quantity and can determine readily the ingredients required.

The algebraic method ordinarily involves no additional calculations to determine the minimum weight of the correcting batch, although in some cases it is not readily apparent which ingredients will be required for restandardization.

Formulas derived algebraically can likewise be used. A few are given later, and the methods of solution by their use illustrated.

Graphic Method of Restandardization.—To illustrate this method, reference will be made to figure 1 (p. 20).

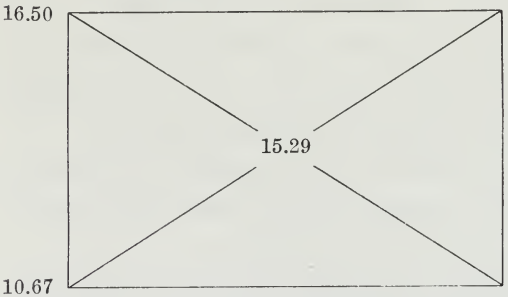
Suppose that F_1 represents the composition of the dairy-products portion of the mix, and A is the desired composition. If a line drawn through the points F_1 and A is extended to intersect line CD at G, then G represents the composition of the milk and cream mixture which, when combined with F_1 in the proper proportion, will give a mix of the desired composition A. If F_2 represented the composition of the off batch, then a mixture of cream and condensed milk would be required for restandardization; whereas if the off batch were of F_3 composition, a mixture of milk and condensed milk would be necessary to adjust it to the desired composition. Since the method of restandardization in all three cases would be the same, only the solution for F_1 will be given.

By referring to figure 1 and reading from the graph, the values representing the compositions of the ingredients designated by the respective points under consideration, we see that:

F_1, the composition of the dairy-products portion of the off batch, contains 16.50 per cent fat and 13.50 per cent milk-solids-not-fat, which is equivalent to 14.03 per cent fat and 11.48 per cent milk-solids-not-fat for the off batch as a whole.

A, the desired composition of the dairy-products portion of the mix, contains 15.29 per cent fat and 12.35 per cent milk-solids-not-fat, which is equivalent to 13.00 per cent fat and 10.50 per cent milk-solids-not-fat in the final mix.

G, the composition of the milk and cream mixture to be combined with the dairy-products portion of the off batch (F_1), contains 10.67 per cent fat and 8.00 per cent milk-solids-not-fat, as can be seen by referring to figure 1. Thus



4.62 = unit parts of off batch

1.21 = unit parts of milk and cream mixture

Then $4.62 + 1.21 = 5.83$ = total unit parts of mixture testing 15.29 per cent fat.

Hence $\frac{4.62}{5.83} = 0.79245$ = fraction that the off batch will be of the final weight of the mix; and

$\frac{100.00}{0.79245} = 126.19$ = pounds of the final mix.

It is more convenient to combine the two previous steps as follows:

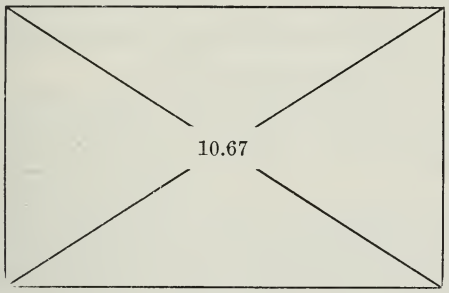
$\frac{5.83}{4.62} \times 100 = 126.19 =$ pounds of final mix.

Consequently $126.19 - 100.00 = 26.19 =$ pounds of minimum correcting batch.

Then $0.146 \times 26.19 = 3.82 =$ pounds of sugar required; and
$0.004 \times 26.19 = 0.10 =$ pounds of gelatin required.

Hence $26.19 - (3.82 + 0.10) = 22.27 =$ pounds of milk and cream mixture required.

Next determine the pounds of milk and cream respectively required to make 22.27 pounds of the mixture containing 10.67 per cent fat. Thus



7.07 = unit parts of cream (D)

22.33 = unit parts of milk (C)

Then $7.07 + 22.33 = 29.40 =$ total unit parts of milk and cream mixture testing 10.67 per cent fat; and

$$\text{pounds of cream} = \frac{7.07}{29.40} \times 22.27 = 5.36.$$

Therefore the pounds of milk $= 22.27 - 5.36 = 16.91$.

Tabulated check 13 verifies these calculations.

TABULATED CHECK 13

Ingredients used	Weight	Fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Off Batch	100.00	14.03	11.48	40.51
Sugar	3.82	.....	.....	3.82
Gelatin	0.10	.....	.....	0.10
Cream testing 33.0 per cent fat	5.36	1.77	0.32	2.09
Milk testing 3.6 per cent fat	16.91	0.61	1.47	2.08
Totals supplied	126.19	16.41	13.27	48.60
Totals required	126.19	16.40	13.25	48.57

On a minimum-weight basis, unless the graphic method is employed, it is ordinarily somewhat easier to restandardize a mix containing the fat and milk-solids-not-fat in excess of that desired, than to restandardize one with the fat and serum solids both too low, or with one of them too low and the other too high. If both are too high, the problem consists

in getting the fat and milk-solids-not-fat in the desired ratio and then diluting with water to the correct final weight, after considering the required amount of nonmilk products. One can accomplish this aim by using only one dairy product, either a fat concentrate, such as cream, or a serum-solids concentrate, such as condensed milk, according to the problem at hand. If the worker desires to use more than one dairy product in any one case, he can then restandardize without adding water. Both types of problem will be illustrated in the following sections.

Formula Method of Restandardization.—The following formulas may be used for calculating the amount of cream or condensed milk required for restandardization where the fat and milk-solids-not-fat are both too high and where water will be used for part of the dilution.

First case: Where the fat is not proportionately so high as the milk-solids-not-fat, obviously cream or some other fat concentrate will be necessary for restandardization.

Let W = weight of off batch with f fraction fat and s fraction milk-solids-not-fat.

X = weight of cream added to correct the off batch with a fraction fat and b fraction milk-solids-not-fat.

$$R_1 = \frac{\text{per cent fat}}{\text{per cent milk-solids-not-fat}} \text{ in the final mix.}$$

Then $$R_1 = \frac{fW + aX}{sW + bX}.$$

Solving this equation for X, we obtain

$$X = \frac{R_1 sW - fW}{a - R_1 b}.$$

This may be expressed as follows:

$$\text{Pounds of cream} = \frac{\left[\begin{array}{c}R_1 \times \text{pounds of}\\ \text{milk-solids-not-}\\ \text{fat in off batch}\end{array}\right] - \left[\begin{array}{c}\text{pounds of}\\ \text{fat in}\\ \text{off batch}\end{array}\right]}{\left[\begin{array}{c}\text{fraction of fat}\\ \text{in cream}\end{array}\right] - \left[\begin{array}{c}R_1 \times \text{fraction of}\\ \text{milk-solids-not-}\\ \text{fat in cream}\end{array}\right]}.$$

Problem: Suppose 100 pounds of a mix testing 13.0 per cent fat and 41.0 per cent total solids must be restandardized on a minimum-weight basis to contain 12.0 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.5 per cent gelatin. The products available for restandardization are sugar, gelatin, water, and cream testing 30.0 per cent fat.

The gelatin and sugar contents in the original mix are assumed to be correct. This enables one to calculate the milk-solids-not-fat content of the off batch.

The steps in calculating the solution to this problem are given below:

Calculate the percentage of milk-solids-not-fat in the off batch by subtracting from the percentage of total solids the percentages of fat, sugar, and gelatin. Thus

$41.0 - (13.0 + 15.0 + 0.5) = 12.5 =$ per cent of milk-solids-not-fat in the off batch.

Calculate the ratio of fat to serum solids in the desired mix. Thus

$$R_1 = \frac{12.0}{10.0} = 1.2.$$

Calculate the pounds of cream required for restandardization by using the formula given above for this case. Thus

$$\text{Pounds cream} = \frac{(1.2 \times 12.5) - 13.0}{0.30 - (1.2 \times 0.063)} = 8.91.$$

Calculate the final weight to which the mix must be diluted in either of the following ways:

Divide the total pounds of fat supplied, by the fraction of fat desired in the corrected mix; or divide the total pounds of milk-solids-not-fat by the fraction of milk-solids-not-fat desired in the corrected mix.

Since the method of calculation is the same in both cases, only the first way will be used here. Thus

$$\begin{aligned} 100 \quad &\times 0.13 = 13.00 = \text{pounds of fat.} \\ 8.91 &\times 0.30 = 2.67 = \text{pounds of fat.} \\ \text{Total supplied} &= 15.67 = \text{pounds of fat.} \end{aligned}$$

$$\text{Hence } \frac{15.67}{0.12} = 130.58 = \text{final weight of mix in pounds.}$$

The additional weights of sugar and gelatin required are calculated in the usual manner. Thus

$$\begin{aligned} 130.58 - 100.0 &= 30.58 = \text{pounds of correcting batch.} \\ \text{Then } 30.58 \times 0.15 &= 4.59 = \text{pounds of sugar; and} \\ 30.58 \times 0.005 &= 0.15 = \text{pounds of gelatin.} \end{aligned}$$

The weight of water for the final dilution equals the difference between the total weight desired and the weight of the ingredients required to supply the necessary solids in the mix. Thus

30.58 pounds $- (8.91 + 4.59 + 0.15)$ pounds $= 16.93$ pounds of water.

These calculations are verified in tabulated check 14.

Second case: Where the milk-solids-not-fat are not proportionally so high as the fat, condensed milk or some other serum-solids concentrate will be necessary to adjust the mix to the desired composition.

In this case let

$$R_2 = \frac{\text{per cent milk-solids-not-fat}}{\text{per cent milk fat}} \text{ in the final mix.}$$

Then we can derive the following formula for the pounds of condensed milk in essentially the same way as the formula for cream in the preceding case.

$$\text{Pounds of condensed milk} = \frac{\left[\begin{array}{c}R_2 \times \text{pounds} \\ \text{of fat in} \\ \text{off batch}\end{array}\right] - \left[\begin{array}{c}\text{pounds of milk-} \\ \text{solids-not-fat} \\ \text{in off batch}\end{array}\right]}{\left[\begin{array}{c}\text{fraction of milk-} \\ \text{solids-not-fat in} \\ \text{condensed milk}\end{array}\right] - \left[\begin{array}{c}R_2 \times \text{fraction} \\ \text{of fat in} \\ \text{condensed milk}\end{array}\right]}.$$

Problem: Suppose 100 pounds of a mix testing 15.0 per cent fat and 41.0 per cent total solids must be restandardized on a minimum-weight basis to contain 12.5 per cent fat, 10.0 per cent milk-solids-not-fat, 14.5

TABULATED CHECK 14

Ingredients used	Weight	Fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Off batch testing 13.0 per cent fat and 41.0 per cent total solids	100.00	13.00	12.50	41.00
Cream testing 30.0 per cent fat	8.91	2.67	0.56	3.23
Sugar	4.59	.....	.....	4.59
Gelatin	0.15	.....	.....	0.15
Water	16.93	.....	.....	.....
Totals supplied	130.58	15.67	13.06	48.97
Totals required	130.58	15.67	13.06	48.97

per cent sugar, and 0.5 per cent gelatin. The gelatin and sugar contents in the original batch are assumed to be correct. The milk product available for restandardization is condensed milk testing 8.0 per cent fat and 20.0 per cent milk-solids-not-fat.

The steps in the restandardization by the serum-point, or formula, method are as follows:

Calculate the percentage of milk-solids-not-fat in the off batch by difference as in the first case (p. 39). Thus

$$41.0 \text{ per cent} - (15.0 + 14.5 + 0.5) \text{ per cent} = 11.0 \text{ per cent.}$$

The ratio of milk-solids-not-fat to fat is calculated for the desired mix. Thus

$$R_2 = \frac{10.0}{12.5} = 0.80.$$

Calculate the pounds of condensed milk required by using the formula given for this case. Thus

$$\text{Pounds of condensed milk} = \frac{(0.80 \times 15.0) - 11.0}{0.20 - (0.80 \times 0.08)} = 7.35 \text{ pounds.}$$

Ascertain, by either of the methods suggested in the first case, the final weight to which the mix must be diluted. The calculations here will be made on the basis of the fat content. Thus

$$\begin{aligned} 100 \quad &\times 0.15 = 15.00 = \text{pounds of fat.} \\ 7.35 &\times 0.08 = 0.59 = \text{pounds of fat.} \\ \text{Total fat in mix} &= 15.59 = \text{pounds of fat.} \end{aligned}$$

$$\text{Hence the final weight of mix in pounds} = \frac{15.59}{0.125} = 124.72.$$

TABULATED CHECK 15

Ingredients used	Weight	Fat	Serum solids	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Off batch testing 15.0 per cent fat and 41.0 per cent total solids	100.00	15.00	11.00	41.00
Evaporated milk testing 8.0 per cent fat and 20.0 per cent milk-solids-not-fat	7.35	0.59	1.47	2.06
Sugar	3.58	.....	.....	3.58
Gelatin	0.12	.....	.....	0.12
Water	13.67	.....	.....	.....
Totals supplied	124.72	15.59	12.47	46.76
Totals required	124.72	15.59	12.47	46.77

The additional weights of sugar and gelatin required are calculated as usual. Thus

$$\begin{aligned} 124.72 - 100.00 &= 24.72 = \text{pounds of correcting batch.} \\ \text{Then } 24.72 \times 0.145 &= 3.58 = \text{pounds of sugar; and} \\ 24.72 \times 0.005 &= 0.12 = \text{pounds of gelatin.} \end{aligned}$$

The weight of water for the final dilution is calculated by difference, as in the first case. Thus

$$24.72 \text{ pounds} - (7.35 + 3.58 + 0.12) \text{ pounds} = 13.67 \text{ pounds} = \text{weight of water required.}$$

Tabulated check 15 verifies the preceding calculations.

Algebraic Method of Restandardization.—When the algebraic method is used for restandardization on a minimum-weight basis without dilution with water, the principle is the same whether both fat and serum solids in the off batch are too high or too low, or whether one is too high and the other too low. The first step is to determine which of the dairy products available will be required for restandardization. Then set up simultaneous equations using the unknowns and solve as previously indicated.

Problem: Suppose, for example, we wish to restandardize 100 pounds of mix testing 11.0 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.4 per cent gelatin so that the final batch will con-

tain 13.0 per cent fat, 10.5 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.4 per cent gelatin. The dairy products available are cream testing 40.0 per cent fat, milk testing 4.0 per cent fat, and condensed skim milk testing 32.0 per cent serum solids.

Since the mix is low in both milk fat and milk-solids-not-fat content, obviously, only cream and condensed skim milk will be required for the restandardization, as far as dairy products are concerned. The unknowns are as follows:

X = weight of cream required.
Y = weight of condensed skim milk required.
Z = minimum weight of the final mix.

The equations will then be:

(1) $0.846Z = X + Y + 84.60$ (milk-products equation).
(2) $0.105Z = 0.054X + 0.32Y + 10.00$ (milk-solids-not-fat equation).
(3) $0.130Z = 0.40X + 11.00$ (fat equation).

The solution follows; subtracting:

(4) $27.072Z = 32X + 32Y + 2{,}707.2$		(Equation 1 × 32)
(5) $10.500Z = 5.4X + 32Y + 1{,}000.0$		(Equation 2 × 100)
(6) $16.572Z = 26.60X +$	1,707.2	(Equation 4 — equation 5)
(7) $0.623Z = X +$	64.18	(Equation 6 ÷ 26.6)
(8) $0.325Z = X +$	27.50	(Equation 3 ÷ 0.4)
(9) $0.298Z =$	36.68	(Equation 7 — equation 8)

Solving equation 9 for Z,

$$Z = \frac{36.68}{0.298} = 123.09.$$

Substituting this value of Z in equation 3 and solving for X,

$0.13 \times 123.09 = 0.4X + 11.0$;

$$X = \frac{16.00 - 11.00}{0.4} = 12.50.$$

Substituting these values in equation 1 and solving for Y,

$0.846 \times 123.09 = 12.50 + Y + 84.60$;

$Y = 104.13 - (84.60 + 12.50) = 7.03.$

The nonmilk products required are calculated as usual. Thus

$123.09 - 100.0 = 23.09$ = pounds of correcting batch.
Then $23.09 \times 0.15 = 3.46$ = pounds of sugar; and
$23.09 \times 0.004 = 0.09$ = pounds of gelatin.

Tabulated check 16 verifies these calculations.

The following suggestions are offered as a guide in ascertaining which of the dairy products available will be needed in restandardizing mixes by this method.

If the off batch is low in both fat and milk-solids-not-fat, a fat concentrate and a milk-solids-not-fat concentrate will be required. Obviously milk or skim milk would not be helpful in raising either the fat or the milk-solids-not-fat in the mix.

If both fat and milk-solids-not-fat are high in the off batch, then milk or skim milk will be required, and either a fat concentrate or a milk-solids-not-fat concentrate will probably also be needed. Where water is used after adjusting the fat and milk-solids-not-fat to the desired ratio with either a fat or milk-solids-not-fat concentrate, only one milk product will be needed.

TABULATED CHECK 16

Ingredients used	Weight	Fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Off batch testing 11.0 per cent fat and 36.4 per cent total solids	100.00	11.00	10.00	36.40
Cream testing 40.0 per cent fat	12.50	5.00	0.68	5.68
Condensed skim milk testing 32.0 per cent milk-solids-not-fat	7.03	.....	2.25	2.25
Sugar	3.46	.....	.....	3.46
Gelatin	0.09	.....	.....	0.09
Totals supplied	123.08	16.00	12.93	47.88
Totals required	123.09	16.00	12.92	47.88

If the fat content is high and the milk-solids-not-fat content is low, we could make the required adjustment by adding a serum-solids concentrate alone or, most likely, by using it in combination with milk or skim milk.

If the off batch is low in milk-fat content and high in milk-solids-not-fat content, a fat concentrate is required either alone or in combination with one of the other milk products.

Although in the borderline cases, inspection does not always indicate which products to use, the exact combination of ingredients would, as shown, be apparent if the graphic method of restandardization were employed.

CALCULATION OF OVERRUN AND WEIGHT OF FOOD SOLIDS PER GALLON

The ice-cream manufacturer is concerned with getting a desired increase in volume of ice cream as compared with the volume of mix and flavoring used. One should carefully control this increase, called *overrun,* in order to produce a uniform product and to meet certain legal standards. According to the California Agricultural Code (see p. 3),

ice cream must contain 1.6 pounds of food solids per gallon; ice milk 1.3 pounds per gallon. Obviously, then, the minimum weight per gallon of ice cream or ice milk will depend upon the percentage of food solids, and this in turn regulates the maximum per cent overrun that can be taken in any given case. The situation is evident if one refers to the following formulas for calculating per cent overrun:

$$\text{Per cent overrun} = \frac{\text{gallon of ice cream made} - \text{gallons of mix and flavoring used}}{\text{gallons of mix and flavoring used}} \times 100; \text{ or}$$

$$\text{per cent overrun} = \frac{\text{weight of 1 gallon of flavored mix} - \text{weight of 1 gallon of ice cream}}{\text{weight of 1 gallon of ice cream}} \times 100.$$

Any unit of volume can be used in these calculations provided the same unit is used throughout a given set of calculations.

Various types of overrun testers can be used as guides throughout the freezing operation.

The following examples will illustrate the use of the formulas just given:

Suppose that from 105 gallons of mix and flavoring 200 gallons of ice cream were obtained. Then, according to the first formula,

$$\text{Per cent overrun} = \frac{200 - 105}{105} \times 100 = 90.48.$$

Again, suppose the net weight of 20 gallons of ice cream proves to be 96 pounds and the mix from which the ice cream was made weighed 9.2 pounds per gallon. Then, according to the second formula,

$$\text{Per cent overrun} = \frac{9.2 - 4.8}{4.8} \times 100 = 91.67.$$

If we desire to use 20 gallons instead of 1 as the basis of calculation, the final result is the same. Thus

$$\text{Per cent overrun} = \frac{184 - 96}{96} \times 100 = 91.67.$$

Ordinarily the terms *food solids* and *total solids* are used synonymously in the ice-cream industry, since with very few exceptions all solids in ice-cream mixes are food solids. Thus, to ascertain the food solids per gallon in ice cream, we may calculate as follows:

$$\left(\begin{matrix}\text{Weight of 1 gallon} \\ \text{of ice cream}\end{matrix}\right) \times \left(\begin{matrix}\text{Fraction of total} \\ \text{solids it contains}\end{matrix}\right) = \left(\begin{matrix}\text{Weight of food solids in} \\ \text{1 gallon of ice cream}\end{matrix}\right).$$

To illustrate the use of this formula, also to show the relation of overrun to food solids per gallon in ice cream, consider this problem. Suppose that an ice-cream inspector found the following in a licensed

ice-cream plant: composition of ice cream by analysis, 13 per cent fat and 38.6 per cent total solids; weight of ice-cream mix used 9.2 pounds per gallon; net weight of 40 gallons of ice cream 190 pounds, which is equivalent to 4.75 pounds per gallon for the ice cream.

Using these data, he could calculate (1) the per cent overrun in the ice cream; (2) the food solids per gallon in the ice cream; (3) the minimum legal weight per gallon of the ice cream in California; and (4) the maximum per cent overrun possible for legal ice cream according to the California standard.

The calculations are made in the order listed.

$$(1)\ \text{Per cent overrun} = \frac{9.2 - 4.75}{4.75} \times 100 = 93.68.$$

$$(2)\ \text{Food solids per gallon} = 4.75 \times 0.386 = 1.83.$$

Before proceeding further we might well consider formulas for calculating minimum weight per gallon and maximum per cent overrun to meet the food-solids-per-gallon requirement.

The minimum weight per gallon can be calculated as follows:

$$\text{Minimum pounds per gallon to meet California standard for ice cream} = \frac{1.6}{\text{fraction of total solids in ice cream}}$$

Or if the maximum overrun is already known, then it can be calculated as follows:

$$\text{Minimum pounds per gallon to meet California standard for ice cream} = \frac{\text{weight of 1 gallon of mix in pounds}}{\text{maximum legal per cent overrun} + 100} \times 100.$$

The maximum legal per cent overrun can be calculated as follows:

$$\text{Maximum legal per cent overrun according to California standard} = \frac{\left(\begin{array}{c}\text{pound of food solids}\\ \text{in 1 gallon of flavored}\\ \text{mix used}\end{array}\right) - 1.6}{1.6} \times 100.$$

Or if the minimum legal weight per gallon of ice cream is already known, then the maximum legal per cent overrun can be calculated as follows:

$$\text{Maximum legal per cent overrun according to California standard} = \frac{\left(\begin{array}{c}\text{weight of 1 gallon}\\ \text{of flavored mix}\end{array}\right) - \left(\begin{array}{c}\text{minimum legal}\\ \text{weight of 1}\\ \text{gallon of ice cream}\end{array}\right)}{\text{minimum legal weight of 1 gallon of ice cream}} \times 100.$$

Thus it can be seen that the basis of calculations for any particular problem will, ordinarily, depend upon the data available.

Using one of these formulas in each case, let us proceed with the calculations necessary for solving the problem at hand.

(3) Minimum pounds per gallon to meet California standard for ice cream $= \frac{1.6}{0.386} = 4.145.$

(4) Maximum legal per cent overrun according to California standard $= \frac{9.2 \times 0.386 - 1.6}{1.6} \times 100 = 121.95.$

TABLE 6

MIX COMPOSITION IN RELATION TO DESIRED FOOD SOLIDS PER GALLON AND MAXIMUM PER CENT OVERRUN

Total solids content of mix	To supply 1.6 pounds of food solids per gallon		To supply 1.8 pounds of food solids per gallon	
	Minimum weight per gallon	Maximum overrun	Minimum weight per gallon	Maximum overrun
per cent	*pounds*	*per cent*	*pounds*	*per cent*
36.0	4.444	104.8	5.000	82.0
37.0	4.324	110.5	4.865	87.1
38.0	4.211	116.1	4.737	92.1
39.0	4.103	121.8	4.615	97.2
40.0	4.000	127.5	4.500	102.2
41.0	3.902	133.2	4.390	107.3

Fortunately most of the ice cream made in California contains more than the minimum food solids per gallon. Ordinarily, other things being equal, the product is better when ice cream contains 1.8 pounds of food solids per gallon than when it contains only 1.6 pounds. Certain states already have adopted 1.8 pounds of food solids per gallon as a minimum standard.

Ice-cream mixes will usually weigh 9.0 to 9.2 pounds per gallon. The exact weight in any case can be determined. It is assumed in the calculations given in table 6 that 9.1 pounds equals the weight per gallon of all mixes. This assumption will modify slightly the calculated per cent overruns in some cases from their true values, but will not affect the calculated values for minimum weights per gallon required to supply the stipulated pounds of food solids per gallon.

USEFUL INFORMATION

Volume

1 liquid quart . . =946.33 milliliters
=32 fluid ounces
=256 fluid drams
=57.75 cubic inches
=2 pints

1 liter . . . . . =0.2642 liquid gallon
=1.0567 liquid quarts
=1,000 milliliters
=61.025 cubic inches
=0.9081 dry quart

1 milliliter . . . . =0.03381 U.S. liquid ounce

1 U.S. liquid oz. . =29.573 milliliters

1 cubic inch . . . =16.387 cubic centimeters

1 cubic foot . . . . =0.02832 cubic meter
=29.922 liquid quarts

1 cubic yard . . . =0.765 cubic meter

1 cubic meter . . =35.314 cubic feet

1 gallon . . . . . =4 liquid quarts

1 barrel . . . . . =31½ gallons

Energy and Pressure

1 horsepower . . . =746 watts
=76 kilogram-meters per sec.
=550 foot-pounds per second

1 BTU . . . . . =777.97 foot-pounds
=252 gram-calories

1 kilowatt-hour . =2,575,000 foot-pounds
=3,412 British thermal units

1 atmosphere . . . =14.7 pounds per square inch at sea level
=33.947 ft. of water at 62° F
=30 ins. of mercury at 62° F
=29.92 ins. of mercury at 32° F
=1.033 kilograms per square centimeter

1 lb. per sq. in. . . =0.0703 kilograms per square centimeter

Weight

1 avoirdupois lb. =16 avoirdupois ounces
=453.59 grams
=7,000 grains

1 kilogram . . . . =2.2046 avoirdupois pounds
=2.6792 Troy pounds
=1,000 grams

1 gram . . . . . =0.03527 avoirdupois ounce
=0.03215 Troy ounce

1 Troy pound . . =12 Troy ounces
=373.24 grams
=0.823 avoirdupois pound

1 Troy ounce . . . =31.103 grams

Length

1 inch . . . . . =2.54 centimeters

1 meter . . . . . =39.37 inches
=100 centimeters
=1,000 millimeters

1 millimeter . . . =1,000 microns

1 foot . . . . . =0.3048 meter
=12 inches

1 mile . . . . . =5,280 feet
=1,760 yards
=1,609.3 meters

Area

1 square yard . . =0.83613 square meter
=9 square feet

1 square meter . . =1.196 square yards
=10.764 square feet

1 square inch . . . =6.452 square centimeters

1 square mile . . . =640 acres

Temperature

1 degree C. . . . =9/5 degree F

1 degree F. . . . =5/9 degree C

0 degree C. . . . =32 degrees F

100 degrees C. . . =212 degrees F

Specific Gravity

For liquids heavier than water:

$$\text{Specific gravity}\frac{60^\circ\text{ F}}{60^\circ\text{ F}} = \frac{145}{145 - \text{degrees Baumé}}, \text{ or}$$

$$\text{Degrees Baumé} = 145 - \frac{145}{\text{specific gravity}\dfrac{60^\circ\text{ F}}{60^\circ\text{ F}}}$$

For liquids lighter than water:

$$\text{Specific gravity}\frac{60^\circ\text{ F}}{60^\circ\text{ F}} = \frac{140}{130 + \text{degrees Baumé}}, \text{ or}$$

$$\text{Degrees Baumé} = \frac{140}{\text{specific gravity}\dfrac{60^\circ\text{ F}}{60^\circ\text{ F}}} - 130$$

TABLE 7

MILK AND CREAM WEIGHTS: APPROXIMATE WEIGHTS PER GALLON AT VARIOUS TEMPERATURES*

Temperature	Skim milk	4 per cent milk	20 per cent cream	25 per cent cream	30 per cent cream	35 per cent cream	40 per cent cream
° *F*	*pounds*	*pounds*	*pounds*	*pounds*	*pounds*	*pounds*	*pounds*
40	8.660	8.610	8.540	8.500	8.470	8.450	8.420
45	8.655	8.605	8.525	8.485	8.450	8.425	8.395
50	8.650	8.600	8.510	8.470	8.430	8.400	8.370
55	8.645	8.595	8.495	8.450	8.410	8.375	8.345
60	8.640	8.590	8.480	8.435	8.390	8.350	8.320
65	8.635	8.585	8.460	8.420	8.370	8.330	8.295
70	8.630	8.580	8.450	8.400	8.350	8.310	8.270
75	8.625	8.570	8.435	8.385	8.330	8.290	8.250
80	8.620	8.565	8.420	8.370	8.315	8.270	8.230
85	8.610	8.560	8.405	8.355	8.295	8.250	8.210
90	8.600	8.550	8.390	8.340	8.280	8.230	8.190
95	8.590	8.535	8.380	8.325	8.265	8.215	8.170
100	8.580	9.520	8.370	8.310	8.250	8.200	8.150
105	8.570	8.510	8.360	8.300	8.240	8.190	8.140
110	8.555	8.500	8.350	8.290	8.230	8.175	8.125
115	8.542	8.485	8.335	8.280	8.215	8.165	8.110
120	8.530	8.475	8.325	8.270	8.205	8.155	8.100
125	8.517	8.465	8.315	8.260	8.195	8.145	8.090
130	8.505	8.450	8.300	8.250	8.185	8.130	8.070
135	8.490	8.440	8.290	8.240	8.170	8.120	8.065
140	8.480	8.430	8.280	8.230	8.160	8.110	8.050
145	8.470	8.420	8.270	8.220	8.150	8.100	8.045

* From: Pirtle, T. R. A handbook of dairy statistics. p. 10. U.S. Department of Agriculture, Washington, D. C. 1928.

TABLE 8

APPROXIMATE COMPOSITIONS AND WEIGHTS PER GALLON OF INGREDIENTS USED IN ICE-CREAM MIXES*

Ingredients	Fat	Milk-solids-not-fat	Sugar	Total solids	Weight per gallon
	per cent	*per cent*	*per cent*	*per cent*	*pounds*
Water	0.0	0.0	0.0	0.0	8.3
Skim milk	0.02	9.0	0.0	9.0	8.6
Milk	3.5	8.6	0.0	12.1	8.6
Milk	4.0	8.8	0.0	12.8	8.6
Milk	5.0	9.1	0.0	14.1	8.6
Cream	20.0	7.2	0.0	27.2	8.5
Cream	25.0	6.7	0.0	31.7	8.4
Cream	30.0	6.3	0.0	36.3	8.4
Cream	35.0	5.8	0.0	40.8	8.4
Cream	40.0	5.4	0.0	45.4	8.3
Evaporated milk	7.9	18.1	0.0	26.0	8.9
Condensed skim milk	0.3	26.7	0.0	27.0	9.3
Condensed skim milk	0.4	31.6	0.0	32.0	9.4
Sweetened condensed skim milk	0.3	27.7	42.0	70.0	11.3
Sweetened condensed whole milk	8.0	20.0	43.0	71.0	10.9
Skim-milk powder	1.0	96.0	0.0	97.0	....
Whole-milk powder	28.0	69.0	0.0	97.0	....
Butter, unsalted	83.0	0.7	0.0	83.7	....
Butter, salted	80.5	0.7	0.0	81.2	....
Butter oil	100.0	0.0	0.0	100.0	7.8
Granulated sugar	0.0	0.0	100.0	100.0	....
Corn sugar (dextrose)	0.0	0.0	92.8	92.8	....
Invert-sugar sirup	0.0	0.0	80.0	80.0	11.8
Corn sirup	0.0	0.0	45.0	82.0	12.0
Dried corn-sirup solids (Fro-dex)	0.0	0.0	55.0	96.5	....
Enzyme converted sirup (Sweetose)	0.0	0.0	55.0	83.0	12.0
Honey	0.0	0.0	76.0	83.0	11.9
Gelatin	0.0	0.0	0.0	90.0	....
Cocoa (powder)	25.0	0.0	0.0	97.0	....
Chocolate liquor	50.0	0.0	0.0	97.0	....
Egg, liquid whole	11.5	0.0	0.0	24.0	8.6
Egg, liquid yolk	32.0	0.0	0.0	51.0	8.4
Egg, powdered whole	40.0	0.0	0.0	95.0	....
Egg, powdered yolk	60.0	0.0	0.0	96.0	....

* Compiled from various sources; most determinations of weight per gallon are for room temperature, or temperatures at which products are ordinarily stored.

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